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A DESCRIPTION OF POCKET AND MAGAZINE CASES OF *MATHEMATICAL DRAWING INSTRUMENTS*;

IN WHICH IS EXPLAINED
THE USE OF EACH INSTRUMENT,
AND PARTICULARLY OF THE SECTOR AND PLAIN SCALE,
IN THE
SOLUTION OF A VARIETY OF PROBLEMS;
LIKEWISE,
THE DESCRIPTION, CONSTRUCTION, AND USE, OF
GUNTER's SCALE.

ILLUSTRATED WITH COPPER-PLATES.

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c64

BRITISH MUSEUM
2 JUN 56

PREFACE.

WHEN a treatiſe makes its appearance in public, whether an original or a compilation, cuſtom has eſtabliſhed it as a rule for the author to ſay ſomething by way of preface—to inform the reader what gave birth to the undertaking, or what he may expect to meet with therein.

In conformity then to this rule, I ſhall briefly premiſe, that, having been in the habit of teaching the Mathematics for a few years, I have had frequent occaſion, in the courſe of the time, to obſerve that the deſcription, conſtruction, and uſe of a Caſe of Mathematical Drawing Inſtruments would make a tract not unintereſting to the public, particularly to thoſe perſons whoſe ſituation in life requires a practical knowledge of the Mathematics, ſuch as naval and military characters, artiſts, architects, &c.

Indeed a Caſe of Mathematical Drawing Inſtruments has always been eſteemed an article of ſuch general utility, that people in almoſt every ſphere of life are in poſſeſſion of them; yet I may venture to aſſert, that their uſe beyond that of deſcribing a circle with the compaſſes, laying off a given length from a ſcale of equal parts, with a few other ſimple problems, generally ſpeaking, is unknown; nay, I have met with ſeveral who have gone through the firſt ſix books of Euclid, without being able to raiſe a perpendicular, or to draw a line parallel to a given line.

This may be owing, in ſome meaſure, to the very few publications that are extant on the ſubject of mathematical inſtruments, the greateſt part of which are out of print, and the reſt are too voluminous and expenſive for general uſe.

In the preſent treatiſe I have confined myſelf to thoſe inſtruments which are commonly met with in Portable Caſes, diſtinguiſhed by the names of Pocket and Magazine Caſes of Mathematical Inſtruments, with the addition of the deſcription and uſe of Gunter's Scale, which, in general, accompanies the others. The uſe of each inſtrument I have endeavoured to explain in the ſolution of ſeveral problems, to which they may be applied; and, as the lines on the ſector are chiefly uſed in trigonometry, I have given the ſolution of all the caſes in plane and oblique triangles, with their application to the meaſurement of heights and diſtances, by that inſtrument.—I flatter myſelf it will be found ſerviceable in ſchools, where every boy immediately on entering upon any branch of practical mathematics, ſuch as menſuration, trigonometry, land-ſurveying, &c. is ſure to be preſented with a Caſe of Inſtruments from the maſter, who perhaps is as little acquainted with their extenſive uſe as the pupil.'

Upon the whole, general utility was my plan; and, if I have been happy enough to ſucceed in the execution thereof, my end in every other reſpect is fully anſwered.

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A

DESCRIPTION, &c.

THE Science of Geometry is become much more extenſive than the etymology of the name implies, which in its literal ſignification is nothing more than "To meaſure the earth;" a name given by the Egyptians to the art by which they diſtinguiſhed, and marked out their lands, after an inundation of the river Nile; but now it extends to all mathematical figures. Geometry may be divided into two parts, theoretical and practical; the former treating of the properties of lines and figures, and the latter of their application to the uſeful purpoſes of life. The following pages are intended to explain the conſtruction and uſe of ſuch inſtruments as have been found moſt neceſſary towards the performance of practical Geometry, previous to which, it may not be amiſs to lay down the following definitions of the firſt book of Euclid's Elements.

DEFINITIONS.

1. A point has no parts.

2. A line is length, without breadth.

3. Points are the extremities of a line.

4. A right line is that which lies evenly between its extremities.

5. A ſuperficies is that which hath length and breadth only.

6. Lines are the extremities of a ſuperficies.

7. A plane ſuperficies is that in which any two points being taken, a right line betwen them ſhall lie wholly in that ſuperficies.

8. A right-lined angle is the inclination of two lines to one another, prolonged till they meet in a point.

9. The lines which form the angle are called the legs of the angle.

10. The point in which the legs meet one another is called the vertex, or angular point.

Note. An angle is generally expreſſed by three letters, whereof the middle one is placed at the vertex, and the other two ſomewhere at the legs. Fig. 1. plate 3.

11. When a right line, ſtanding upon a right line, ſhall make the adjacent angles (ABC and ABD) equal, each of the equal angles is called a

right angle, and the line which ſtands upon the other line, is called a perpendicular. Fig. 2, pl. 3.

12. An obtuſe angle is greater than a right angle.

13. An acute angle is leſs than a right angle.

14. A plane figure is a plane ſuperficies, terminated on every ſide by one or more lines.

15. A circle is a plane figure, contained under one line, which is called the circumference; to which all right lines drawn from a certain point within the figure, are equal.

16. This point is called the center of the circle.

17. The diameter of a circle is a right line drawn through the center, and terminated each way in the circumference.

18. The radius of a circle is a right line drawn from the center to the circumference.

19. A ſemicircle is a figure contained by the diameter, and that part of the circumference which is intercepted by the diameter.

20. A rectilinear figure is a plane ſuperficies, terminated on every ſide by right lines.

21. A triangle is a plane ſuperficies, contained under three right lines.

22. An equilateral triangle is that whoſe three ſides are equal.

23. An iſoſceles triangle is that which hath two of its ſides equal.

24. A ſcalene triangle is that which hath three unequal ſides.

25. A right-angled triangle is that which hath one right angle.

26. An obtuſe-angled triangle is that which hath one obtuſe angle.

27. An acute-angled triangle is that which hath three acute angles.

28. Parallel right lines are ſuch as, being drawn on the ſame plane, and produced, *ad infinitum*, will never meet.

29. A quadrilateral figure is a right lined figure contained between four right lines.

30. A parallelogram is a quadrilateral figure, whoſe oppoſite ſides are parallel.

31. A ſquare is a quadrilateral figure, equilateral and rectangled.

32. Rectilinear figures which have more than four ſides, are called Polygons.

ADDITIONAL DEFINITIONS.

33. The circumference of every circle is ſuppoſed to be divided into 360 equal parts, called degrees.

34. A ſemicircle contains 180 degrees.

35. The meaſure of an angle is that arc, or portion of a circle, which is contained between the

two legs; the angular point being the centre of the circle.

36. A right angle is equal to 90 degrees.

37. In the circle, BDC (fig. 9, pl. 3.) DE is called the Sine; DC the Chord; GC the Tangent; and AG the Secant, of the arc, or portion of the circumference DC; which arc is likewiſe the meaſure of the angle DAC.

Moreover IC is a diameter, AC the radius, and AB the ſine of 90 degrees, which is always equal to the radius.

The Deſcription and Uſe of a Caſe of Mathematical Drawing Inſtruments.

Inſtruments for drawing mathematical figures are generally ſold in ſets, and fitted into a caſe, which, when ſmall and portable, is called a Pocket Caſe; but when large, and a variety of inſtruments inſerted, a Magazine Caſe of Inſtruments.

A complete Pocket Caſe generally contains the following Inſtruments:

1. A pair of large compaſſes with a plain point, an ink point, a pencil point, and ſometimes a dotting point.

2. A pair of ſmall plain compaſſes.

3. A drawing pen, which contains likewiſe a protracting pin.

4. A pair of bow compaſſes.
5. A parallel ruler.
6. A protractor, ſometimes ſemicircular, but moſt commonly of the form of a parallelogram, one ſide of which is a plane ſcale.
7. A ſector.
8. A black lead pencil.

In a Magazine Caſe are commonly the following Inſtruments:

1. A pair of large compaſſes, with the ſame moveable points as thoſe in the common caſe, and an additional piece to prolong the leg of the compaſſes, ſo as to enable them to deſcribe circles of a greater radius, transfer longer diviſions, &c.
2. A pair of hair compaſſes.
3. A pair of bow compaſſes.
4. A pair of triangular compaſſes.
5. A ſector.
6. A parallel ruler with croſs bars.
7. A protractor.
8. A pair of proportional compaſſes.
9. A drawing pen with a protracting pin.
10. A knife, file, and ſcrew driver.
11. A pair of gunner's calipers; and, if required,
12. A ſet of water colours.

DESCRIPTION OF THE POCKET CASE.

1. *Of the Plain Compaſſes.*

This inſtrument is ſo very well known by every artiſt and mechanic, that a minute deſcription would be ſuperfluous. It is repreſented at fig. A. pl. 1.

Plain compaſſes are uſed in meaſuring lines or diſtances, deſcribing circles, conſtructing figures, &c.

PROBLEM I.

To draw a line of any propoſed length.

Draw with the pencil an indefinite line, and opening the compaſſes till the diſtance between the points be equal to the propoſed length, ſet one foot thereof on one end of the line, and with the other make a point in the line, which will mark out the required length.

PROBLEM II.

To deſcribe a circle having the diameter given.

Draw a line equal to half of the given diameter, which will be the radius; take the length of this line between the points of the compaſſes, and placing one point at the end of the line, ſweep the other point round the fixed one, and a circle will be formed, whoſe diameter will be equal to that propoſed.

PROBLEM III.

To deſcribe a circle having the circumference given.

The diameter of a circle is to the circumference as 7 to 22, nearly: therefore ſay,

As 22 : 7 : : propoſed circumference : the diameter; which being found, the circle may be deſcribed as in the preceding problem.

PROBLEM IV.

To biſect or divide a given right line into two equal parts. Fig. 3, pl. 3.

Let the given line be AB. On the point A, with any extent greater than half the given line, deſcribe the arc, *wx*; on B, with the ſame extent, deſcribe another arc, *yz*, which will cut the former in two points; lay a ruler upon theſe points of interſection, and along the edge thereof draw a line, which will cut AB into two equal parts in the point C, as was propoſed.

PROBLEM V.

To raiſe a perpendicular from a given point in a given line. Fig. 4, pl. 3.

Let the given line be AB, and the given point C. On C as a center, with any extent, draw the ſemicircle *xy*; on *x*, with the radius *xy*, deſcribe an arc *n*; on *y*, with the radius *yx*, croſs the former arc *n*, through which point of interſection a right line drawn from C will be the perpendicular required.

PROBLEM VI.

To let fall a perpendicular from a given point to a given right line. Fig. 5, pl. 3.

Let the given line be AB, and the given point C. On C as a center, deſcribe an arc cutting the line in E and F; on E and F as two centers, and with the diſtance EF, deſcribe arcs cutting each other, on which point of interſection and the point C, lay a ruler, and by the edge thereof draw CD, which will be a perpendicular to the given right line.

2. *Of the large Compaſſes with moveable Points.* *Fig.* B, *pl.* 1.

The uſe of theſe compaſſes is the ſame with the laſt; but as in drawing figures, plans, architecture, &c. it is frequently neceſſary to make uſe of auxiliary circles, lines, &c. intended only for a temporary purpoſe, there is in one leg of theſe compaſſes a groove or ſocket to receive different points according to the purpoſe deſigned; for inſtance, if a permanent a permanent arc is intended to be drawn, the ink point (fig. C.) muſt be uſed; if a temporary one, the pencil point, which may afterwards be taken out by a piece of Indian rubber: this pencil point may be uſed in drawing the circular arcs in the foregoing problems; it is repreſented at fig. D. The dotting point has a ſmall indented wheel at the end, and is intended for drawing dotted lines, but is now ſeldom found in a caſe of inſtruments.

3. *Of the Drawing Pen and Protracting Pin.*

This is a ſteel pen, fixed to a braſs rod or handle of a convenient length, for drawing ſtraight black lines by the edge of a rule, inſtead of uſing a common pen. In the middle part of the handle is a ſcrew, and when unſcrewed, a fine ſteel point draws out for terminating lines, marking degrees from the protractor, &c. Fig. E, E.

The drawing pen has two ſides, whoſe points may be opened or cloſed by an adjuſting ſcrew, in order to draw the line fine or coarſe, as may be requiſite. The points muſt be opened to receive the ink, and then ſcrewed together.

4. *Of the Bow Compaſſes. Fig.* F, *pl.* 1.

Theſe are a pair of ſmall compaſſes, one leg of which is a ſteel point, and the other a drawing pen with a joint. Their uſe is to draw ſmall circles or arcs where thoſe of a larger ſort would be found inconvenient. The head of the bow compaſſes is ſo conſtructed, that it will roll round with great eaſe between the fingers.

5. *Of the Parallel Ruler. Fig.* M, *pl.* 2.

This inſtrument conſiſts of two ſtraight rulers, connected together by two equal and parallel pieces of braſs or other metal, moving on the points which faſten them to the two rulers. This common parallel ruler is objectionable on this account, that when opened out, the moveable part ſlides away obliquely from the fixed one inſtead of riſing perpendicularly to it, therefore in the better kind of caſes is contained a parallel ruler with croſs bars to remedy this inconvenience.

In this inſtrument the two ſtraight pieces of wood are joined together by two bars, moving on a common center, which center is their interſecting point; two of their ends likewiſe turn on centers or pins, which fix them to the rulers, and the other two ends ſlide in grooves when the rulers open and ſhut.

Parallel rulers are contrived to facilitate the drawing of parallel lines, a problem which is continually occurring in practical geometry.

PROBLEM.

Through a given point to draw a right line parallel to a given right line.

Let C be a given point, and AB a given right line, it is required to draw a line through the point C that ſhall be parallel to AB, Fig. 6, pl. 3.

OPERATION.

Lay the ruler ſo that the lower edge may exactly coincide with the given line, and hold the lower part firm in that poſition; then move the other part till the edge thereof touch the point C, along which a line being drawn, will be parallel to the given line AB.

6. *Of the Protractor and Plane Scale.* *Fig.* N, *pl.* 2.

This inſtrument is a plane ruler in the form of a parallelogram, made of ivory or braſs. On its two ſides are found, moſt commonly, the following lines.

On one ſide, as X.

A diagonal ſcale of equal parts.

A protracting ſcale, to lay down and meaſure angles.

On the other ſide, as Y.

Six ſcales of equal parts, ſimply divided.

A ſcale of chords, marked Cho, or C.

Another ſcale of equal parts, marked EP.

Another ſcale of chords, marked Cho.

A line of ſines, marked S, or Sin.

A line of tangents, marked T, or Tan.

A line of ſecants, marked S, or Sec. which generally begins where the line of ſines ends.

Beſides theſe, there is ſometimes a third line of chords, under which is a line of rhumbs, and under that a line of longitude; the uſe of all which will be explained after we have deſcribed the manner in which they are conſtructed.

To conſtruct a diagonal ſcale of equal parts.

1. Draw eleven parallel lines at equal diſtances from each other, and divide the uppermoſt of theſe lines into as many equal parts as the intended ſcale to be conſtructed ſhall contain; from each of theſe diviſions croſs the eleven parallels by lines perpendicular to them, and conſequently parallel to each other. 2. Subdivide the firſt of theſe diviſions into 10 equal parts, both on the uppermoſt and loweſt parallel. 3. Draw oblique lines from the firſt perpendicular or O above, to the firſt ſubdiviſion below; from the firſt ſubdiviſion above, to the ſecond below; from the ſecond above, to the third below; and ſo on till the laſt, namely, from the ninth above

to the tenth below. By theſe oblique or diagonal lines each of the ſmall diviſions are again ſubdivided into 10 equal parts, conſequently the firſt primary ſpace is now divided into 10 times 10, or 100 equal parts; for as each of the firſt ſubdiviſions is one tenth of a primary diviſion, ſo each parallel upwards is divided into tenths of the firſt ſubdiviſions by the oblique lines. If, therefore, one of the primary diviſions be called an unit, each of the firſt diviſions will be one tenth of an unit, and each of the ſecond ſubdiviſions one hundreth of an unit; if one of the primary diviſions be ten, the firſt ſubdiviſion will be an unit, and each of the ſecond ſubdiviſions one tenth of an unit, and ſo on.

To conſtruct a Scale of equal Parts, ſimply divided.

Draw two or three parallel lines, and divide each of them into as many equal parts as the ſcale is intended to contain—ſubdivide the firſt of theſe into 10 equal parts, and the ſcale is finiſhed.

Several of theſe ſcales are laid down on the inſtrument we are now deſcribing; they are diſtinguiſhed by numbers placed at the left hand end of the ſcale, which numbers ſhew how many parts of an inch each line is divided into: for inſtance, the firſt primary diviſion on every ſcale is ſubdivided into 10 equal parts. Now if 20 of theſe ſubdiviſions are contained in an inch, it is marked a ſcale of 20; if 30 of theſe parts are equal to an inch, it is marked a ſcale of 30; and ſo on. If one of theſe ſubdiviſions be called an unit, a primary diviſion will be ten; and if the ſubdiviſions be each ten, the primary diviſions will be each one hundred. If a ſmall diviſion be made to repreſent one tenth of an unit, then a primary diviſion will be an unit, &c.

And, moreover, as feet and inches are the denominations of meaſure moſt in uſe among artificers, the firſt primary diviſion on theſe ſcales of equal parts, is generally divided duodecimally, or into twelve parts, by an upper line; ſo that, if one of the primary diviſions be taken as one foot, each of theſe ſubdiviſions will be one inch.

To conſtruct the protracting ſcale.—On that ſide of the plane ſcale which contains the diagonal ſcale of equal parts, along the two ends and upper margin are two parallel ſpaces, divided each into 180 parts, numbered 10, 20, 30, 40, &c. from right to left, and 10, 20, 30, 40, &c. the contrary way from left to right—the lines which form theſe diviſions all converge towards one point, marked juſt at the middle of the lower margin of the ruler, which is plain and undivided. If on this point as a centre, a circle or any number of circles be deſcribed, and if from thence the diviſions on the three margins of the protracting ſcale be prolonged till they meet the circumferences of theſe circles, half of each of them will be divided into 180 equal parts, being the number of degrees into which every ſemicircle is divided, as mentioned in the definitions.

To conſtruct the Lines of Chords, Sines, Tangents, &c. Fig. 7, pl. 3.

With any radius or opening of the compaſſes, deſcribe the circle ABCD, which divide into four quadrants or quarters, by the diameters AB and CD croſſing each other at right angles; each of theſe quadrants will contain 90 degrees. Then,

1. To project a line of chords.—Divide the arc of one of the quadrants, as BC, into 9 equal parts, then will each part contain 10 degrees, and mark the points of diviſion with 10, 20, 30, 40, &c. to 90, beginning at C. Then, if right lines be drawn from C to theſe ſeveral diviſions, each right line ſo drawn will be the chord of its reſpective arc. Draw therefore the whole chord of 90 degrees BC, and on the point C as a centre, transfer with your compaſſes the ſeveral diſtances C 10, C 20, C 30, &c. to the line BC, which will mark out the points 10, 20, 30, 40, &c. on BC, and divide it into a ſcale of chords corresponding to the radius OC.

2. To conſtruct a line of ſines. From the ſeveral diviſions of the arc BC (divided as before) let

fall perpendiculars upon the radius OC; or, which is the ſame thing, draw lines parallel to BO: theſe lines will fall upon the radius OC in the points 10, 20, 30, 40, &c. which muſt be numbered from O towards C for right ſines, and from C towards O for verſed ſines; thus will the line OC be divided into a line of ſines, correſponding with the radius OC.

3. To conſtruct a line of tangents.—From the extremity C of the radius OC, erect the perpendicular CG, or, which is the ſame thing, draw CG parallel to OB; then, having the arc CB divided into 9 equal parts, as before, lay a ruler on the centre O, and on each of the ſeveral diviſions 10, 20, 30, &c. of the arc BC, and draw ſtraight lines cutting the perpendicular CG in the points 10, 20, 30, &c. CG will then be divided into a line of tangents correſponding to the radius OC.

4. To draw a line of ſemitangents.—From the extremity of the diameter D, apply a ruler upon each of the 9 diviſions of the arc BC, and along the edge thereof draw the right lines D 10, D 20, D 30, &c. interſecting the radius (or ſine of 90 degrees) BO; the line BO thus divided will be a line of ſemitangents, correſponding to the radius OC.

5. To conſtruct a line of ſecants.—The lines which are drawn from the centre O through the ſeveral diviſions of the arc BC, to the line of

tangents

tangents CG, are the ſecants of the different arcs through which they paſs; therefore place one foot of the compaſſes in the centre O, and transfer the ſeveral diviſions O 10, O 20, O 30, &c. on the line CG, to 10, 20, 30, &c. on the line OB prolonged to F; ſo will BF be divided into a line of ſecants, correſponding to the radius OC.

Note. Theſe lines in this figure are only conſtructed to every 10 degrees of the quadrant, but by ſubdividing each of theſe diviſions on the arc BC into 10 equal parts, and lines drawn as above directed, any of the ſcales may be conſtructed to every ſingle degree.

6. To project a line of rhumbs.—Divide the arc of the quadrant BD into 8 equal parts, in the points 1, 2, 3, 4, &c. Then draw the whole chord of the quadrant DB, and placing one foot of the compaſſes in D, transfer the ſeveral diviſions D 1, D 2, D 3, &c. from the arc to the line DB; ſo will the line DB be a line of rhumbs, or the points of the mariner's compaſs, each point being 11 degrees 15 minutes.

7. To conſtruct a line of longitude.—Divide the radius OA into 60 equal parts (60 miles being contained in every degree of longitude under the equator), and mark every 10 with its proper number; from theſe diviſions let fall perpendiculars upon the arc AD, or, which is the ſame thing, draw the lines from each diviſion on OA parallel to OD;

draw the right line or chord AD, and ſetting one foot of the compaſſes on A, transfer the diſtances from A to the ſeveral diviſions on the arc to the line AD, ſo will AD be divided into a line of longitude correſponding to the radius OC.

Note. The radius, the chord of 60 degrees, the ſine of 90, and the tangent of 45 degrees, are all alike, as may be ſeen in the figure, which is univerſally the caſe.

Theſe are the lines moſt commonly found on plane ſcales, but ſome of the old ones contain beſide theſe a line of latitude, and another of hours, conſtructed for the purpoſe of dialling; but, as they are of very little uſe, and ſeldom met with, I ſhall omit their conſtruction, and proceed to illuſtrate the uſe of the lines already deſcribed, by a few examples to each.

The Uſe of the ſimply divided Scale of equal Parts.

Scales of equal parts are uſed in laying down plans of buildings, ſurveys of eſtates, making lines of any propoſed length, &c. each diviſion being called 1 foot, 1 yard; 10 feet, 10 yards; 100 feet, 100 yards, &c. as is found moſt convenient for the ſize of the figure to be conſtructed.

EXAMPLE I.

Let it be required to lay down a line whoſe length ſhall repreſent 4,5, 45, 450, or 4500 yards, furlongs, miles, or any other meaſure, from the ſcale marked 30.

Draw with a pencil an unlimited right line, and taking the propoſed ſcale of equal parts, ſet one foot of the compaſſes on the diviſion marked 4, and open the other to the fifth ſubdiviſion, reckoning towards the left hand: this extent of the compaſſes applied upon the given right line will determine its length, which may repreſent either $4\frac{5}{10}$, 45, 450, or 4500 yards, furlongs, miles, or any other meaſure.

EXAMPLE II.

To draw a line that ſhall repreſent 6 feet 9 inches.

Draw an unlimited right line as before; and, having made choice of a convenient ſcale, place one point of the compaſſes on 6 among the primary diviſions, and extend the other to the ninth ſubdiviſion on the upper line (which diviſions repreſent inches); the compaſſes thus extended, applied upon the right line, will determine the propoſed length, namely, 6 feet 9 inches.

The Uſe of the Diagonal Scale of equal Parts.

The uſe of this ſcale is the ſame with thoſe above mentioned, only it is more general, and diſtances may be meaſured by it more accurately than by thoſe ſimply divided.

EXAMPLE I.

Take off 649 from the diagonal ſcale of equal parts.

Set one foot of the compaſſes upon the primary diviſion marked 6, and on the ninth parallel line which croſſes the ſame; then open the other leg to the point where the fourth oblique line cuts the ninth parallel line, that extent will be the required diſtance; for the 6 primary diviſions will repreſent 600, the 4 ſubdiviſions 4 tens or 40, and the 9 parallels cut by the fourth oblique line will be 9 units. But this extent or opening of the compaſſes not only repreſents 649, but alſo 64,9; 6,49; ,649, &c. for if the 6 primary diviſions be 6 tens, the 4 firſt ſubdiviſions will be 4 units, and the 9 ſecond ſubdiviſions 9 tenths of an unit: or, if the 6 primary diviſions be taken as 6 tenths of an unit, the 4 firſt ſubdiviſions will be 4 hundredths of an unit, and the 9 ſecond ſubdiviſions will be 9 thouſandth parts of an unit.

EXAMPLE II.

Set off 12,4 yards from a diagonal ſcale.

Place one foot of the compaſſes upon 12 on the loweſt parallel, and open the other to the fourth ſubdiviſion on the ſame parallel; but if the ſcale does not contain 12 primary diviſions, place one foot of the compaſſes upon the fourth parallel of the diviſion marked 1, and open the other along the ſame parallel to the point where it croſſes the ſecond oblique line, ſo will you have in the extent of the compaſſes 1 ten, 2 units, and 4 tenths, or 12,4, the diſtance required to be taken off.

EXAMPLE III.

To lay down a line of 4 feet 9 inches.

In this caſe and in all others, where the parts of the integer are not decimally divided, it will be neceſſary, before they can be laid off from the diagonal ſcale, to reduce them to the decimal of their integer. Thus 9 inches are $\frac{9}{12}$ of a foot, or ,75 of a foot when turned into a decimal, therefore the number to be taken off the ſcale is 4,75. Set one foot of the compaſſes upon the fifth parallel where it cuts the fourth primary diviſion, and extend the other along the ſame parallel to the point where it meets the ſeventh oblique line; the diſtance between the points of the compaſſes will repreſent 4 feet 9 inches.

Example IV.

To meaſure a line already drawn by the diagonal ſcale.

Take the length of the line between the points of the compaſſes, and placing one foot upon ſuch a primary diviſion that the other may fall among the diagonals, ſlide both points upwards till each of them fall upon an interſection on the ſame parallel, that is, one point being on a vertical or primary diviſion, and the other on a diagonal or oblique line; ſuppoſe, for inſtance, one foot ſtood on the fifth primary diviſion and on the ſixth parallel of the ſame, and that the other reſted on the ſame parallel where it interſected the ninth oblique line, the meaſure of the line would then be ,596, 5,96 59,6, or 596 miles, furlongs, yards, &c. according to the meaſure in which the given line was meant to be expreſſed.

Example V.

Having three ſides of any triangle given to conſtruct the ſame from the diagonal ſcale. Fig. 8, pl. 3.

Let the ſide AB be 6,45, AC 3,81, and BC 4,78 chains; it is required to conſtruct the triangle.

Draw an unlimited right line, and laying off 6,45 (the longeſt ſide) from the diagonal ſcale, apply it to the line, and mark the extremities thereof with A and B. 2dly, From the ſame ſcale take another ſide as 3,81, and with that extent of the compaſſes, ſetting one foot on the point A, with the other deſcribe a ſmall arc towards C. 3dly, Take the third ſide 4,78 from the ſame ſcale of equal parts, and with that extent and one foot on B deſcribe a ſecond arc, which will interſect the firſt in the point C; lay a ruler from C to A, and from C to B, and by the edge thereof draw the lines CA and CB, ſo will the figure ABC be the triangle required. Thus a triangular piece of land, whoſe three ſides were 6,45, 3,81, and 4,78 chains, would be repreſented on paper by the figure ABC in its true form and dimenſions.

The Uſe of the Protracting Scale.

This part of the plane ſcale is uſed to lay down angles of any given magnitude, and to meaſure thoſe already drawn. As all right-lined figures take their form according to the angles under which their terminating lines or boundaries are drawn, the uſe of this ſcale forms no inconſiderable part of practical geometry, which, though not the moſt accurate, is yet by far the moſt expeditious method of laying down or meaſuring angles.

EXAMPLE I.

At the point A to conſtruct an angle that ſhall be equal to 45 degrees. Fig. 10, pl. 3.

Place the center-mark of the protractor upon the point A, ſo that at the ſame time the edge of the inſtrument may coincide or lay even with the line A*c*: then, beginning to reckon at the left hand end of the ſcale, count to the forty-fifth diviſion in the order of 10, 20, 30, &c. and there make a point with the protracting pin (already deſcribed); then laying a ruler upon A and this point, draw the line A*b*, and the angle *b*A*c* will contain 45 degrees.

EXAMPLE II.

To make an angle at the point A in the line A*d*, equal to $60\frac{1}{2}$ degrees. Fig. 11, pl. 3.

Lay the center of the protractor upon the point A, and at the ſame time let the edge thereof coincide with the line A*d*; then counting from left to right, make a mark on the paper directly half way between the 60th and 61ſt degree: through this mark draw A*c*; then will the angle *c*A*d* be equal to $60\frac{1}{2}$ degrees.

Note. Theſe two laſt being leſs than 90 degrees each, are acute angles.

EXAMPLE III.

To make an angle at the point A in the line A*f*, equal to 118¾ degrees. Fig. 12, pl. 3.

Lay the centre of the protractor upon A, and let the edge coincide with the line A*f*; then count from right to left till you come to the 118th diviſion and three quarters of another diviſion farther, and make a mark with the protracting pin; through A, and this mark, draw the line A*g*, and the angle *g*A*f* will be equal to 118¾ degrees.

EXAMPLE IV.

To find the meaſure of an angle already drawn, ſuppoſe *b*A*c*. Fig. 10, pl. 3.

Lay the protractor upon A*c* with the centre upon A, then mark the point of diviſion upon the ſcale beyond which the line A*b* directly proceeds, and the degrees intercepted between the right hand end of the ſcale, and that diviſion which coincides with the line A*b* will be the meaſure of the angle *b*A*c*. If the line A*b* is too ſhort to extend beyond the edge of the protractor, it will be neceſſary to prolong it.

Note. In very common caſes of inſtruments, inſtead of the protractor we have now deſcribed, there is a ſemicircular one of braſs; but, as the principle and method of uſing them are exactly the ſame, a particular deſcription will be unneceſſary.

The Uſe of the Line of Chords.

By a line of chords angles may be laid down or meaſured the ſame as with the protractor, as will appear by the following examples:

EXAMPLE I.

At the point A in the line A*b*, to lay down an angle by the line of chords equal to 45 degrees. Fig. 10, pl. 3.

Take in your compaſſes from the line of chords the diſtance from 1 to 60, and with that extent and one foot in the point A deſcribe the arc *cb* of any length; then from the ſame ſcale of chords take the diſtance from 1 to 45, which apply upon the arc deſcribed from *b* to *c*; laſtly, through A and *c* draw a right line A*c*, and the angle *c*A*b* will be equal to 45 degrees.

Note. The chord of 60 degrees being equal to the radius, muſt always be uſed to deſcribe the arc, on which the number of degrees contained in the angle to be made, is to be applied.

Example II.

To meaſure an angle already laid down; for inſtance, the angle *b*A*c*. Fig. 10, pl. 3.

From the ſcale of chords take in the compaſſes the chord of 60 degrees, and ſetting one foot in A, deſcribe the arc *bc* as before; then take in the compaſſes the diſtance from *b* to *c*, and placing one foot at the beginning of the line of chords, let the other fall along the line, and it will ſhew the meaſure of the angle, which in this caſe is 45 degrees. But, if the angle to be conſtructed or meaſured exceed 90 degrees, it will be neceſſary to halve it, and turn that diſtance twice over upon the arc.

The Uſe of the Line of Rhumbs.

This line is nothing more than a line of chords differently divided, the one being marked at every 10 degrees from 1 to 90, and the other at every 11¼ degrees from 1 to 8, which diviſions form the points of the mariner's compaſs. The uſe of this line is to lay down the angle of a ſhip's courſe in navigation.

Example.

EXAMPLE.

Suppoſe a ſhip's courſe was E. N. E. or 6 points from the meridian, it is required to draw the ſame. Fig. 13, pl. 3.

Draw the line AB to repreſent the meridian or north and ſouth line, from B with the extent of 60 degrees from the line of chords, deſcribe the arc AC, upon which, from the point A lay the diſtance from 1 to 6, or 6 points taken from the line of rhumbs, from A to C; draw BC, which will repreſent the courſe of a ſhip ſailing from B in the direction of E. N. E. or 6 points from the meridian AB.

Likewiſe, by the aſſiſtance of a line of rhumbs, the card of a mariner's compaſs may be drawn. For if a circle be deſcribed with the chord of 60 degrees, and through the center be drawn a meridian line or diameter, whoſe extremities ſhall repreſent the north and ſouth points, and if from each of theſe extremities the ſeveral diviſions of the line of rhumbs be transferred upon the circumference of the circle, from north to eaſt and weſt, and from ſouth to eaſt and weſt, the whole circle will be divided into 32 equal parts, each part containing 1 point of the compaſs, or 11¼ degrees.

The Uſe of the Line of Longitude.

This line being placed contiguous to a ſcale of chords of the ſame radius, and numbered in a contrary direction, ſerves to ſhew, by inſpection, how many miles are contained in a degree of longitude in any parallel of latitude; this is done by reckoning the given latitude upon the line of chords, and immediately oppoſite, on the line of longitude, is ſeen the number of miles that compoſe a degree of longitude in that parallel.

EXAMPLE I.

What number of miles are contained in a degree of longitude in the latitude of 60 degrees.

Look for 60 on the line of chords, and directly oppoſite to it, on the line of longitude, you will find 30, which is the number of miles that make a degree of longitude in the parallel of 60 degrees.

EXAMPLE II.

Required the number of miles in a degree of longitude in the latitude of 80 degrees.

Look for 80 on the line of chords, and oppoſite thereto, on the line of longitude, ſtands 10, the number of miles in a degree of longitude in the latitude of 80 degrees.

The line of ſines is chiefly uſed in ſpherical trigonometry, and in the orthographic projection of the ſphere.

The lines of tangents, ſemitangents, and ſecants, are likewiſe uſed in ſpherical triangles, and to find the poles of projected circles, &c.

Before we proceed to the next article, it may not be amiſs to give the deſcription and uſe of an inſtrument well known by name, and in moſt people's hands who are any way concerned in the practice of drawing plans, &c. I mean Gunter's Scale, which though it does not make a part of a caſe of mathematical inſtruments, yet moſt commonly accompanies one.

Deſcription and Uſe of Gunter's Scale.

This inſtrument is a ſtraight ruler, two feet in length.

On one of its ſides are contained,

1. A line of 24 inches.
2. Two diagonal ſcales of equal parts.
3. A ſcale of equal parts marked L.

4. A ſcale of equal parts, marked P.
5. A ſcale of equal parts, marked Lea.
6. A line of chords, marked Cho.
7. A line of rhumbs, marked Rhum.
8. A line of ſines, marked Sin. or Sines.
9. A line of tangents, marked Tan.
10. A line of ſemitangents, marked S. tan.
11. A ſecond line of chords at the right hand end.
12. A line of longitude, correſponding to the above line of chords, marked M. L.
13. A ſecond rhumb line to correſpond likewiſe with the above line of chords, marked Rum.

The uſe and conſtruction of all the lines above-mentioned, are exactly the ſame in every reſpect with thoſe already deſcribed on the plane ſcale.

On the other ſide are,

1. A logarithmic ſine rhumb, marked S. R.
2. A log. tangent rhumb, marked T. R.
3. A log. line of numbers, marked N. or Num.
4. A log. line of ſines, marked Sines.
5. A log. line of verſed ſines, marked V. S.
6. A log. line of tangents, marked Tan.
7. A line of meridional parts, marked Mer.
8. A line of equal parts, marked Eq. P.

Of the Line of Numbers.

This line is a table of logarithms laid off upon a ſtraight line, by which, and a pair of compaſſes, all operations in multiplication, diviſion, proportion, menſuration of figures, &c. may be performed with great eaſe and tolerable accuracy, provided the numbers to be worked with be not too great. The line begins to be numbered at the left hand end of the ſcale, where there is a braſs point, and proceeds in a regular ſeries of 1, 2, 3, &c. to 1 again, where there is a ſecond braſs point; and here begins another ſeries the ſame as the firſt, which continues to 10 at the end of the ſcale. Now if the firſt ſeries of numbers from 1 to 10 be accounted units, the ſecond ſeries from 1 to 10 will be tens; if the firſt ſeries be called tens, the ſecond ſeries will be hundreds; and if the firſt ſeries be called tenths of an unit, the ſecond ſeries will be units, and ſo on. Between the primary diviſions, 1, 2, 3, &c. are ſubdiviſions, whoſe value depends upon that of the primary ones: for inſtance, between 3 and 4 we find 10 diviſions; therefore, if we call the primary 3 three units, then each of the intermediate diviſions will be one tenth of an unit; if the 3 be called three tens, or 30, then each of the ſubdiviſions will be an unit; but if the primary diviſion 3 be accounted as three tenths of an unit, then each of the ſubdiviſions will be three hundredth parts of an unit, &c. The ſpace between ſome of the primary diviſions is only ſubdivided into five parts, then in this caſe it is obvious that each of theſe ſubdiviſions will be 2 tenths, 2 units, 2 tens,

&c. according to the value of their preceding primary ones.

Multiplication by the Line of Numbers.

General rule.—Place one leg of the compaſſes upon unity, and extend the other to one of the factors (either the multiplicand or multiplier); that extent turned over in the ſame direction upon the ſcale, will reach from the other factor to the product.

EXAMPLE I. Multiply 4 by 5.

Set one foot of the compaſſes in the braſs point at the firſt unit, and open the other to the number 4; take up the compaſſes, and, without altering the extent, place one leg on 5, and the other will fall upon the number 2 in the ſecond ſeries, which in this caſe will be 20, the 2 in the firſt ſeries being eſtimated at 2 units. The reſult will be the ſame if the extent be taken from 1 to 5, for that applied upon 4 will reach to 20.

EXAMPLE II. Multiply 4 by 8.

Extend the compaſſes from 1 to 4, which extent will reach from 8 to the ſecond ſubdiviſion, after the 3 in the ſecond ſeries of numbers, namely 32; the firſt 3 being reckoned 3 units.

EXAMPLE III. Multiply 2,4 by 2,5.

Extend the compaſſes from 1 to the 4th ſubdiviſion after the 2, and that extent turned over from the 5th ſubdiviſion after 2, will reach exactly to 6, which is 6 units, the compaſſes in this caſe not falling out of the firſt ſeries of numbers; but if it had been required to multiply 24 by 25, the 6 muſt then have been accounted 600.

EXAMPLE IV. Multiply 65 by 50.

Extend the compaſſes from 1 to the firſt 5 (reckoning it 50); apply that extent upon the 5th diviſion after the 6 (namely 65), and the other leg will fall half way between the ſecond and third ſubdiviſions after 3 in the ſecond ſeries; which in this caſe is 3250; for as in the laſt example we may obſerve, that tens multiplied into tens produced hundreds, though the point of the compaſſes did not fall out of the firſt ſeries; therefore in this, and in all others, where the extent reaches from one ſeries into the other, tens multiplied into tens, will produce thouſands; ſo that the 3 will be 3000, the two ſubdiviſions 200, and the half diviſion 50.

Diviſion by the Line of Numbers.

General rule.—Place one leg of the compaſſes upon unity, and the other upon the diviſor; that extent (in a contrary direction to the order of the numbers on the ſcale) will reach from the dividend to the quotient.

EXAMPLE I. Divide 20 by 5.

Set one foot of the compaſſes upon the firſt unit, and open them till the other reſts upon 5; then that extent applied upon 20 (2 in the ſecond ſeries), will reach, in a contrary direction to the order of the ſcale, from thence to 4, which is the quotient.

EXAMPLE II. Divide 32 by 4.

Extend the compaſſes from 1 to the firſt 4; with that extent apply one leg to the ſecond ſubdiviſion after 3 in the ſecond ſeries, the other will reach backwards to 8.

EXAMPLE III. Divide 6 by 2,4.

Extend from 1 to the fourth ſubdiviſion after 2 in the firſt ſeries; that extent applied backwards from 6, will reach to the fifth ſubdiviſion after 2 or 2,5.

Example IV. Divide 3250 by 65.

Extend the compaſſes from 1 to the 5th ſubdiviſion after the firſt 6; that extent will reach from the middle point between the ſecond and third ſubdiviſions after 3 in the ſecond ſeries, backwards to 5 on the firſt, which will be 50.

Proportion by the Line of Numbers.

Rule.—Extend the compaſſes from the firſt term in the proportion to the ſecond; that diſtance applied upon the third term will reach to the fourth.

Example I. Given 2 : 3 :: 6, required the fourth term.

Set one foot of the compaſſes upon 2 on the firſt ſeries, and open the other to 3; that extent applied upon the third term 6, will reach in the ſame direction to 9, which is the fourth term.

Example II. Given three numbers 8 : 5 :: 6 in proportion, required the fourth term.

Extend from 8 in the firſt ſeries backwards to 5; that extent will reach from 6 in the firſt series backwards, to the middle points between the ſeventh and eighth ſubdiviſions after 3, which muſt be read thus, 3 units, 7 tenths and a half, or 5 hundredth parts of an unit; that is, the fourth term will be 3,75.

Of the Line of Sines.

This line is a ſcale of logarithms for every minute of the quadrant, and adapted to the line of numbers; it is of great uſe in trigonometry, for by it, and the line of numbers, all the caſes of plane triangles may be ſolved, as will be ſhewn when we come to practical trigonometry.

Of the Line of Tangents.

This is alſo a ſcale of logarithms proceeding from 1 to 45, and back again from 45 to 80 on the ſame line, but numbered in a contrary direction; it is uſed likewiſe in trigonometry, in conjunction with the line of numbers.

Of the Line of verſed Sines.

This is uſed in ſpherical trigonometry and aſtronomy.

Of

Of the Lines of Sine Rhumbs and Tangent Rhumbs.

Theſe two lines are peculiar to navigation; they are nothing more than the above two lines of ſines and tangents, converted from degrees and minutes into points, half points, and quarter points of the mariner's compaſs. Their uſe will be explained along with the others.

Of the Meridional and Equal Parts.

Theſe two lines anſwer to the meridians and parallels of a mercator's chart, in which all meridians, parallels, and rhumbs, are ſtraight lines, and the degrees of longitude do not converge, but are equally diſtant from one another; and the degrees of latitude increaſe towards the poles in the ſame proportion that the parallel circles on the globe decreaſe. The line of equal parts therefore contains the equal degrees of longitude, and the line of meridional parts the increaſed degrees of latitude. The uſe of theſe lines is to conſtruct a mercator's chart.

7. *Of the Sector.*

There is no mathematical inſtrument of ſuch general and extenſive uſe as the ſector. It takes its name from the tenth definition of the third book of Euclid's Elements of Geometry, which ſays, " A ſector of a circle is that part which is con- " tained between two radii, or ſemidiameters, and " their included circumference;" but the fundamental principle of this inſtrument is derived from the fourth propoſition of the ſixth book of Euclid, which runs thus: " The ſides of equiangled " triangles, which are about the equal angles, are " proportional, and the ſides that are ſubtended by " the equal angles, are homologous." This will be explained when we have deſcribed the inſtrument. —The ſector is formed of two equal ſtraight rules, called legs, which are moveable round a joint; near the centre of this joint is the centre of the inſtrument, to which all ſectoral lines are drawn; for there are many lines upon the ſector which do not proceed from this centre, but theſe are not called ſectoral lines, being ſimilar to thoſe upon a Gunter or plane ſcale.

The ſectoral ſcales or lines, marked on one face of the inſtrument, are,

1. Two ſcales of equal parts, one on each leg, marked L. or Lin. and called the Line of Lines.

2. Two lines of chords, one on each leg, marked C. or Cho.

3. Two lines of ſecants, one on each leg, marked with a ſmall s. or ſec.

4. A line of polygons, marked Pol.

On the oppoſite ſide, the lines are

1. Two lines of ſines, one on each leg, marked S. or Sin.

2. Two lines of tangents, one on each leg, marked T. or Tan. proceeding only to 45.

3. Two other lines of tangents to a leſs radius, to ſupply the deficiency of the former, and proceeding from 45 upwards; theſe lines are marked with a ſmall t.

Theſe ſeveral lines are laid down upon the ſame principle as thoſe of the ſame name upon the plane ſcale, and are placed upon the ſector in ſuch a manner, that every pair of the ſame name or kind ſhall make equal angles at the centre (the line of polygons excepted); therefore, at whatever diſtance the ſector be opened, the angles at the centre formed by each reſpective pair of lines will be equal; that is, the diſtance between 10 and 10 on the line of lines; 90 and 90 on the line of ſines; 60 and 60 on the line of chords; and 45 and 45 on the line of tangents, will remain invariably the ſame at every poſſible opening of the ſector.

The remaining lines on the ſector are thoſe taken from the plane ſcale, and are different on different inſtruments, according to the option of the maker.

Some have on one ſide,

1. A line of inches.
2. A line of latitudes.
3. A line of hours.
4. A line of inclinations of meridians.
5. A line of chords.

On the other ſide,

1. A logarithmic line of numbers.
2. A logarithmic line of ſines.
3. A logarithmic line of tangents.

Theſe three laſt are exactly the ſame as thoſe of the ſame name upon Gunter's ſcale.

Of the Principle upon which Sectoral Lines are drawn.

Let AB and AC repreſent any pair of ſectoral lines, forming an angle BAC with each other; in AC aſſume any point as x, and in AB take Ay equal to Ax, and draw xy; then by the fourth propoſition of the ſixth book of Euclid, already quoted, it will be as Ay to AB, ſo xy to CB; that is, whatever part Ay is of AB, the ſame part will xy be of CB: for inſtance, if AB be 10, and Ax or Ay 5, then, whatever be the line CB, xy will be one half of it. Fig. 14, pl. 4.

And as every pair of ſectoral lines are exactly graduated alike on each leg, from the centre to the extremities of the inſtrument, it follows, that at every opening of the ſector there will be two ſimilar triangles, both of them iſoſceles and equiangled.— Hence if the lines AB or AC be the lines of chords extending to 60°, then will AB or AC alſo be the radius (the chord of 60° and radius being always equal); and ſuppoſe A*x* or A*y* be the chord of any propoſed number of degrees to the ſaid radius AB or AC, then the line *xy* will be the chord of the ſame number of degrees, correſponding to the radius BC.

Note. In operations on the ſector, if a diſtance be taken from the centre along any of the ſectoral lines, as AB, it is called a lateral diſtance; but a diſtance taken from any point in one leg, to the correſponding point of the ſame line on the other leg, as *xy*, is called a parallel diſtance.

The Uſe of the Line of Lines.

The arithmetical rules of multiplication, diviſion, proportion, &c. are performed by theſe ſectoral lines, beſides many other problems, both arithmetical and geometrical.

Multiplication by the Line of Lines.

General rule.—Let the ſector be opened till the parallel diſtance from 10 to 10 be equal to the lateral diſtance of one of the factors; then the parallel diſtance of the other factor, meaſured from the centre upon the line of lines, will be the product.

Example. Multiply 6 by 8.

Extend the compaſſes from the centre of the ſector to 6 on the line of lines, then open the ſector, till the diſtance from 10 to 10 be equal to that extent; take the parallel diſtance from 8 to 8 in the compaſſes, and laying it from the center along the line of lines, it will reach to the eighth ſubdiviſion beyond the number 4, which will be 48.

Note. The ſector being now ſet to the multiplier 6, the parallel diſtance of any other multiplicand may be taken in the compaſſes, and meaſured laterally from the center, by which means the product of any number into 6 may be obtained at once; thus, the parallel diſtance between 4 and 4 will reach to 24, the product of 6 into 4; the parallel diſtance of 5 will extend to 30; the product of 6 into 5; of 6 to 36, &c.

Diviſion by the Line of Lines.

General rule.—Open the ſector till the parallel diſtance of the diviſor is equal to the lateral diſtance of the dividend; then the parallel diſtance of 10, meaſured from the centre on the line of lines, will be the quotient.

EXAMPLE. Divide 48 by 6.

Extend the compaſſes from the center to 48, and make the parallel diſtance of 6 equal to that extent; then take the parallel diſtance of 10 in the compaſſes, and meaſuring from the centre along the line of lines, the extent will reach to 8, which is the quotient.

Proportion by the Line of Lines.

General rule.—Open the ſector till the parallel diſtance of the firſt term be equal to the lateral diſtance of the ſecond term; then the parallel diſtance of the third term, meaſured from the centre along the line of lines, is the fourth proportional.

EXAMPLE. Required a fourth number proportional to 2, 3, 6.

Extend the compaſſes from the centre to 3, and make the parallel diſtance of 2 equal to that extent;

then the parallel diſtance of 6, meaſured from the centre, will extend to 9, the fourth number proportional to the given three.

Note. If it ſhould happen that the parallel diſtance be too great for the opening of the ſector, take ſome aliquot part of the number expreſſing that parallel diſtance, and multiply the reſult of the operation into the number by which the parallel diſtance was divided; for example, let a fourth proportional be found to the numbers 2, 6, and 8. In this caſe we find that the lateral diſtance of the ſecond term 6, is too great for the parallel diſtance of 2; therefore take a part of 6, ſuppoſe $\frac{1}{3}$ of it for a lateral diſtance, namely 2, and make that the parallel diſtance of the firſt term 2; then the parallel diſtance of 8 applied from the centre along the line of lines, reaches to 8, which is $\frac{1}{3}$ part of the fourth proportional.

PROBLEM.

To biſect, or divide into two equal parts, any given ſtraight line.

Take in the compaſſes the extent of the given line, and make the parallel diſtance from 10 to 10 equal to this extent; then the parallel diſtance from 5 to 5 will be one half of the given line. Or if the parallel diſtance from 8 to 8 be made equal to the given line, then the parallel diſtance from 4 to 4 will give one half of the line; likewiſe, if the parallel diſtance from 6 to 6 be made equal to the given line, then from 3 to 3 will be equal to half of it.

PROBLEM.

To divide a given ſtraight line into any propoſed number of equal parts by the line of lines.

Make the parallel diſtance of the points, expreſſing the number of parts into which the given line is to be divided, equal to the extent of the whole line, then the parallel diſtance between 1 and 1 applied upon the given line, will divide it into the number of equal parts required: for inſtance, let a given ſtraight line be divided into 7 equal parts; take the extent of the whole line in the compaſſes, and make the parallel diſtance of 7 equal to that extent, then the parallel diſtance between 1 and 1 will be $\frac{1}{7}$ part of the whole line.—If the given line be too great for the ſector, take $\frac{1}{2}$, $\frac{1}{4}$, &c. of it, which divide as above directed; then the double quadruple, &c. of one of thoſe parts will divide the whole given line into the number of equal parts propoſed.

PROBLEM.

To convert the line of lines into a ſcale of equal parts, containing any number in a given length.

Open the ſector ſo that the given length may become the parallel diſtance between the points, or numbers on the legs expreſſing the number of equal parts, and the ſector is then the required ſcale.

EXAMPLE.

Let the ſector be made into a ſcale of 6 inches, containing 10 equal parts. Open the ſector till the diſtance between 10 and 10 be equal to 6 inches; then the parallel diſtance from 1 to 1, 2 to 2, 3 to 3, transferred upon a line of 6 inches, will mark out the required ſcale.

This is obviouſly nothing more than to divide a given line into any number of equal parts.

PROBLEM.

To divide a given line in the ſame proportion as another given line is divided.

Make the given divided line a lateral diſtance on both legs of the ſector from the centre, and mark the extremity of the ſaid line on each leg. 2. Open the ſector till the parallel diſtance of the extreme points of the divided line be equal to the undivided line. 3. Apply the ſeveral parts of the divided line laterally on the legs of the ſector, and the parallel diſtances of the correſponding diviſions on the legs, applied to the undivided line, will mark out the points of diviſion required.

EXAMPLE.

Let the line CD be divided in the ſame manner as the line AB. Fig. 15, pl. 4.

Place the divided line AB on the line of lines, from A to B on one leg, and from A to C on the other; then open the ſector till the parallel diſtance from B to C (the extremities of the divided line applied upon the legs) be equal to the undivided line CD: let the ſector remain in his poſition, and transfer the parts of the divided line AE, AF, AG, to the ſector, as AE, AF, AG; then the parallel diſtance of EE will give the diviſion CH, the parallel diſtance of FF will give the diviſion CI, and the parallel diſtance of GG will give the diviſion CK; ſo will the whole line CD be divided in the ſame proportion as the given line AB.

PROBLEM.

To divide a given line into two unequal parts that ſhall be to one another in any required proportion.

EXAMPLE.

Let a given line be divided into two parts, ſo that one part ſhall be to the other as 3 to 5. Make

the

the parallel diſtance of 8 (the ſum of 3 and 5) equal to the length of the given line; then the parallel diſtance between 3 and 3 will be one part, and the parallel diſtance between 5 and 5 will be the other part of the given line; and theſe parts will be to one another as 3 to 5.

PROBLEM.

To find the proportion that any number of given lines bear to each other.

Open the ſector till the longeſt of the given lines becomes the parallel diſtance between 10 and 10; then taking the extent of the leſs lines ſeverally in the compaſſes, ſlide the points along both legs of the ſector, till they reſt exactly upon the ſame number on each leg, which number will be the proportion that line bears to the largeſt, which may be called 10, 100, 1000, &c. care being taken to name the ſmaller lines accordingly; thus, if the points reſted upon 7, the proportion between that line and the greateſt would be as 7 to 10, or as 70 to 100; if at 7,5, the proportion would be as 7,5 to 10; or as 75 to 100; or as 750 to 1000, &c.

PROBLEM.

To open the ſector ſo that the line of lines may form a right angle, or be perpendicular to each other. Make the lateral diſtance of 5 equal to the diſtance between 3 on one leg of the ſector, and 4 on the other, and the line of lines will then ſtand at right angles, or perpendicular to each other.

The reaſon of this is, that a triangle whoſe three ſides are 3, 4, and 5, will be a right-angled triangle.

The Uſe of the Line of Polygons.

Any regular polygon, from 4 to 12 ſides, may be readily conſtructed by this line.

PROBLEM.

To conſtruct a polygon whoſe ſide ſhall be of a given length.

EXAMPLE I.

Let a pentagon (or a figure with 5 ſides) be conſtructed whoſe ſides ſhall be 3 inches.

It has been obſerved, that the chord of 60 degrees is equal to the radius; but 60 degrees is the ſixth part of 360 degrees, or the whole circumference of a circle; therefore the radius, or chord of 60 degrees, will always be equal to a ſide of the hexagon (or figure with 6 ſides), inſcribed in a circle. For this reaſon, the radius of the circle in which polygons conſtructed by this line are inſcribed, muſt always be the parallel diſtance between 6 and 6 on the line of polygons. Therefore, in the above example, open the ſector, till the parallel diſtance between 5 and 5 on the line of polygons be equal to 3 inches, the length of the given ſide; then with the parallel diſtance between 6 and 6, as a radius, deſcribe a circle; and the diſtance between 5 and 5 applied to the circumference, will go juſt 5 times upon the ſame, and determine the pentagon required.

EXAMPLE II.

Let it be required to conſtruct a nonogon (or figure with 9 ſides), each of which ſhall be 1$\frac{1}{2}$ yards, feet, inches, &c.

Open the ſector till the parallel diſtance between 9 and 9 in the line of polygons, be equal to 1,5, taken from ſome ſcale of equal parts; then with the parallel diſtance of 6 and 6 as a radius, deſcribe a circle, and the diſtance between 9 and 9 will juſt go 9 times round the circumference, and determine the points of the nonagon.

The Uſe of the Line of Chords.

The uſe of this line is to lay down angles, and to meaſure thoſe already drawn, ſo that it may be made a ſubſtitute for the protractor.

PROBLEM.

To lay down an angle of any propoſed number of degrees. Fig. 16, pl. 4.

EXAMPLE I.

At the point A in the line AB, let it be required to make an angle of 47 degrees. With any convenient opening of the ſector take the parallel diſtance from 60 to 60 on the line of chords, and with that extent, and one foot of the compaſſes in the point A, deſcribe an arc *xy*; then the lateral diſtance between 47 and 47, applied upon the arc from *x* to *y*, will give the point *y*, through which the line AC being drawn, will make an angle BAC equal to 47 degrees, as was required.

EXAMPLE II.

At the point A, in the line AB, let it be required to conſtruct an angle equal to 110 degrees. Fig. 17, pl. 4.

As the ſcale on the ſector is continued no farther than to 60 degrees, when an angle exceeding that number is required to be drawn, as in the preſent caſe, proceed as follows: With any opening of the ſector, take the parallel diſtance between 60

and

and 60, and deſcribe an arc on A, as before; then take half the given number of degrees, namely 55, and apply the parallel diſtance of 55 and 55 twice upon the arc, and through the ſecond point draw the line AC; the angle BAC will contain the number of degrees required. If the angle to be conſtructed exceeds 120 degrees, then take a third of it for a parallel diſtance to be applied upon the arc three times, to obtain the point through which the line AC muſt paſs, to form the propoſed angle with the line AB.

PROBLEM.

To meaſure an angle already drawn.

EXAMPLE I.

Required the meaſure of the angle BAC. Fig. 16, pl. 4.

With any radius or opening of the compaſſes, draw an arc between the two lines including the angle, and open the ſector, ſo that the parallel diſtance between 60 and 60 may be equal to that radius; then take the arc *xy* in the compaſſes, and carry the points thereof along the line of chords, on each leg of the ſector, till they both reſt upon the ſame number of degrees, which number will be the meaſure of the angle, namely, in the preſent caſe 47 degrees.

EXAMPLE II.

Required the meaſure of the angle BAC. Fig, 17, pl. 4.

With any radius or opening of the compaſſes, draw the arc *xy*, and open the ſector, ſo that the parallel diſtance 60 and 60 may be equal to that radius; then take half the arc *xy* (being greater than 60 degrees) in the compaſſes, which apply to the line of chords on each leg, as above directed; the double of the number of degrees where the points reſt, will be the meaſure of the angle.

PROBLEM.

Having the radius of any circle given, to find the length of the chord of any number of degrees, correſponding to that radius.

EXAMPLE.

Let the radius of a circle be 2 inches, required the length of the chord of 40 degrees to that radius.

Open the ſector till the parallel diſtance of 60 and 60 be equal to 2 inches; then the parallel diſtance between 40 and 40 applied to a ſcale of inches, will give the length of the chord of 40 degrees to the radius 2, namely 1,38 inches.

PROBLEM.

To conſtruct regular polygons by the line of chords.

All regular polygons may be conſidered to be inſcribed in circles, whoſe ſides are the chords of ſome aliquot part of the circumference; thus the ſide of a ſquare is the chord of one fourth part of the circumference of a circle; the ſide of a pentagon the chord of one fifth; a hexagon one ſixth; a heptagon one ſeventh, &c.: therefore, as every circle is ſuppoſed to be divided into 360 equal parts, called degrees, if 360 be divided by the number of ſides in the polygon, we ſhall find what part of the circumference is taken up by one ſide of the polygon, which will be as follows:

360 divided by 5, gives 72° for a ſide of the pentagon.
360 ÷ 6, gives 60° for a ſide of the hexagon.
360 ÷ 7, gives 51$\frac{3}{7}$ for a ſide of the heptagon.
360 ÷ 8, gives 45 for a ſide of the octagon.
360 ÷ 9, gives 40 for a ſide of the nonagon.
360 ÷ 10, gives 36 for a ſide of the decagon.
360 ÷ 11, gives 32$\frac{8}{11}$ for a ſide of the undecagon.
360 ÷ 12, gives 30 for a ſide of the dodecagon.

1ſt, To inſcribe a polygon in a given circle.

EXAMPLE.

Let an octagon be inſcribed in the circle BCD. Fig. 18, pl. 4.

From the centre of the circle A, draw any radius, as AB, and at the point A make an angle (by the line of chords) equal to 45 degrees, that being the part of the circumference taken up by the ſide of an octagon, and draw AC; the chord BC will be one eighth part of the circumference, conſequently a ſide of the octagon.

2nd, To conſtruct a polygon having the length of its ſide given.

As every regular polygon conſiſts of a number of iſoſceles triangles, equal to the number of its ſides, whoſe vertices are the centre of the deſcribed circle, and as the angles at the baſe of every iſoſceles triangle are equal (Euclid i. 4.); it follows, that if the angle at the centre be found, as above directed, the angles at the baſe, ACB or ABC, may likewiſe be known; for the three angles of every triangle are equal to two right angles, or 180 degrees. Euclid i. 32.

Thus in a pentagon the angle at the centre is 72 degrees; this ſubtracted from 180, leaves 108 degrees, the ſum of the two remaining angles ACB and ABC; but as theſe two are equal, the half of 108, namely, 54, will be the number of degrees in each; and thus may the angles of any other polygon be found.

EXAMPLE.

Let it be required to conſtruct a decagon whoſe ſide ſhall be 1,75 inches. Fig. 19, pl. 4.

The angle at the centre, or part of the circumference taken up by a decagon, is 36 degrees; this ſubtracted from 180, leaves 144, half of which is 72 degrees, the meaſure of the angle ACB or ABC.

Draw a ſtraight line as CB, and make it equal to 1,75 from any ſcale of equal parts, and at the extremity C, make an angle ACB equal to 72 degrees, and draw an unlimited line CA; at the extremity B, make an angle ABC equal to 72 degrees, and draw the line BA, which will meet the other line CA in the point A. On A as a centre, with the radius BA or CA, deſcribe a circle; then will BC be the tenth part of this circle, which applied upon the circumference 10 times will give the points in the ſame to which ſtraight lines being drawn, the decagon will be conſtructed.

The Uſe of the Lines of Sines, Tangents, &c.

The uſe of theſe lines in the ſolution of trigonometrical problems is very extenſive; they alſo ſerve to determine the length of the ſine, tangent, or ſecant of an arc, containing any number of degrees, &c.

PROBLEM.

The radius of a circle being given to find the length of the ſine, tangent, or ſecant of any propoſed number of degrees, correſponding to that radius.

EXAMPLE.

Let a circle be given whoſe radius is 3 inches; required the length of the ſine, tangent, and ſecant of 36 degrees, correſponding to that radius.

For the ſine.—Open the ſector till the parallel diſtance between 90 and 90 on the line of ſines be equal to 3 inches; then the parallel diſtance between 36 and 36 on the line of ſines, meaſured on a ſcale of inches, will ſhew the ſine of 36 degrees to the radius 3 inches.

For the tangents.—Open the ſector till the parallel diſtance between 45 and 45 on the line of tangents be equal to three inches; then the parallel diſtance of 36 and 36 on the line of tangents, meaſured on a ſcale of inches, will give the tangent of 36 degrees correſponding to the radius 3 inches.

For the ſecant.—Open the ſector till the parallel diſtance between O and O, on the line of ſecants, be equal to 3 inches; then the parallel diſtance of 36 and 36 on the line of ſecants, meaſured on a ſcale of inches, will ſhew the length of the ſeçant correſponding to that radius.

PROBLEM.

The length of the ſine, tangent, or ſecant of any number of degrees being given, to find the length of the radius of a circle correſponding to each of theſe.

EXAMPLE.

Let the length of the ſine, tangent, and ſecant of 31 degrees, be each 1 inch; required the radius correſponding to theſe.

For the radius to the ſine.—Open the ſector till the parallel diſtance between 31 and 31 on the line of ſines be equal to 1 inch; then the parallel diſtance of 90 and 90 on the ſame line, meaſured on a ſcale of inches, will ſhew the length of the radius correſponding to the given ſine.

For the radius to the tangent.—Open the ſector till the parallel diſtance of 31 and 31 on the line of tangents be equal to 1 inch; then the parallel diſtance of 45 and 45 on the ſame line, meaſured on a ſcale of inches, will give the radius to the tangent.

For the radius to the ſecant.—Open the ſector, ſo that the parallel diſtance of 31 and 31 on the line of ſecants may be 1 inch; then the parallel diſtance of O and O on the ſame line, meaſured on a ſcale of inches, will ſhew the length of the radius to the ſecant.

As the great uſe of the lines of ſines, tangents, and ſecants, as well as the logarithmic lines on the Sector and Gunter's ſcale, is beſt exemplified in Trigonometry, I ſhall lay down a few of the moſt ſerviceable problems therein, ſuch as may be applied to the meaſuring of heights and inacceſſible diſtances, and at the ſame time fully explain the uſe of the above lines.

Solution of Problems in right-angled Plane Trigonometry, by the Sector and Gunter's Scale.

In the triangle ABC, the longeſt ſide AC, is called the hypothenuſe, and the other two ſides AB and BC, are called the legs of the triangle.

The right angle ABC, is ſometimes called the radius ſine of 90 degrees, and ſometimes the radius tangent of 45 degrees, being both equal.

When the hypothenuſe is conſidered as radius, then each leg is proportional to the ſine of its oppoſite angle, and the line of ſines muſt be uſed.

When one leg is conſidered as radius, then the hypothenuſe will be proportional to the ſecant, and the other leg to the tangent of the angle oppoſite to that leg; and the lines of tangents and ſecants muſt be made uſe of.

As the three angles of every triangle are equal to 180 degrees, or two right angles, if one of the acute angles BCA, or BAC be known, the other is alſo known, being 90 degrees together.

Any two ſides, or one ſide and an acute angle, being given, the remaining parts of the triangle may be found.

CASE I.

The hypothenuſe AC, and the angle BAC given, to find the legs AB and BC. Fig. 20, pl. 4.

Let the hypothenuſe be 50 yards, and the angle BAC 32° 45′; required the length of the legs AB and BC.

Note. ° ſtands for degrees, and ′ for minutes.

The proportion will be for the leg BC,

As radius ſine, of 90°,
: the hypothenuſe AC, 50.
:: ſine of the angle BAC, 32° 45′,
: the oppoſite leg BC, 27,05.

therefore on the logarithmic line of ſines on the ſector and Gunter's ſcale proceed as follows:

Extend the compaſſes from the firſt term 90, on the line of ſines, to the third term 32° 45′ on the ſame line; apply that extent upon the ſecond term 50 on the line of numbers, and it will reach to the fourth term, namely 27,05 yards, which is the required length of the leg BC.

For the leg AB,

As radius ſine of 90°,
: hypothenuſe AC, 50.
:: ſine of the angle ACB 57° 15′ (by taking 32° 45′ from 90°),
: the oppoſite leg AB 42,05.

therefore, extend the compaſſes from 90° to 57° 15′ on the line of ſines, that extent will reach from 50 on the line of numbers, to 42,05, the length of the leg AB.

By the ſectoral lines.

Make the parallel diſtance of 90 and 90 on the line of ſines equal to the lateral diſtance of 50 on the line of lines; namely, from the centre to 5; then the parallel diſtance of 32° 45′, and 32° 45′ on the line of ſines applied to the center of the ſector, and meaſured along the line of lines, will give the fourth term or leg BC, equal to 27,05 yards.

For the leg AB. With the ſame opening of the ſector take the parallel diſtance between 57° 15′, and 57° 15′, and apply it upon the line of lines, which will give 42,05 yards for the leg AB.

Geometrical

Geometrical conſtruction.

Draw an unlimited line AB, and at the point A with the protractor, or a ſcale of chords, make an angle CAB equal to 32° 45′, and draw the line AC; from any ſcale of equal parts take off 50, and lay it from A to C; from C let fall the perpendicular BC, and the triangle is conſtructed.

Then BC, meaſured on the ſame ſcale of equal parts with AC, will be 27,05 yards, and AB 42,05 yards.

CASE II.

The hypothenuſe AC and one leg, ſuppoſe AB given, to find the angles and the other leg.

Let the hypothenuſe be 50, and the leg AB 42,05; required the leg BC, and the angles BCA and BAC.

For the angle BCA.

As the hypothenuſe AC 50,
: radius ſine of 90°.
:: leg AB 42,05,
: oppoſite angle BCA 57° 15′, which taken from 90° leaves 32° 45′ for the angle CAB.

By

By the logarithmic lines.

Extend the compaſſes from the firſt term 50 to the third term 42,05 on the line of numbers; that extent will reach from 90 to the fourth term, namely 57° 15′ on the line of ſines.

The leg BC may now be found by the firſt caſe, when the angles are known.

By the ſectoral lines.

Make the parallel diſtance of the ſecond term 90° on the line of ſines, equal to the lateral diſtance of the ſecond term 50, on the line of lines; then the lateral diſtance of the third term 42,05 on the line of lines, will be the parallel diſtance of the third term 57° 15′, and 57° 15′, on the line of ſines, for the angle BCA, and this taken from 90°, leaves 32°45′ for the angle CAB.

For the leg BC. The ſector remaining at the ſame opening, the parallel diſtance of the angle CAB 32° 45′ on the line of ſines, applied from the centre upon the line of lines, will give the length of the oppoſite leg BC 27,05.

Geometrical conſtruction.

Draw a right line AB, and make it equal to the given length 42,05, from any ſcale of equal parts;

at the point B erect the perpendicular BC of an unlimited length; take the hypothenuſe AC 50, from the ſame ſcale of equal parts, and with one foot of the compaſſes in A, croſs the line BC with the other, in the point C; lay a ruler from C to A, and by the edge thereof draw the line CA, and the triangle is conſtructed: then, if the angles be meaſured by a protractor, it will be found that BCA meaſures 57° 15′, and BAC 32° 45′; the ſide BC likewiſe from the ſcale of equal parts will be 27,05.

CASE III.

One angle BAC, and one leg, as AB given; to find the hypothenuſe AC, and the other leg BC.

Let the angle BAC be 32° 45′, and the leg AB 42,05; required the length of the hypothenuſe AC, and the leg BC.

For the hypothenuſe AC.

As ſine of the angle BCA 57° 15′ (found from
the other angle),
: the oppoſite leg AB 42,05;
:: radius, ſine of 90°,
: the hypothenuſe AC 50.

By

By the logarithmic lines.

Extend the compaſſes on the line of ſines from 57° 15′ to 90°; that extent applied to 42,05 on the line of numbers, will reach to 50 on the ſame line, for the length of the hypothenuſe.

The angles and the hypothenuſe being now known, the leg BC may be found by the firſt caſe.

By the ſectoral lines.

Open the ſector till the parallel diſtance of the firſt term 57° 15′, be equal to the ſecond term 42,05, taken from the line of lines laterally; then the parallel diſtance of 90 and 90 on the ſines, will meaſure from the centre of the ſector to 50 on the line of lines.

The ſector being thus ſet, if the parallel diſtance of 32° 45′, and 32° 45′ on the line of ſines, namely, the angle BAC, be applied laterally to the line of lines, it will reach from the centre to 27,05, the length of the leg BC.

Geometrical conſtruction.

Draw a right line BA, and make it equal to 42,05 from any ſcale of equal parts; at the point A, with a ſcale of chords or a protractor, make the angle BAC equal to 32° 45′, and draw at pleaſure the line AC; at the point B erect a perpendicular, which will cut the line AC in C, and determine the triangle. Then AC, taken in the compaſſes, and meaſured upon the ſame ſcale of equal parts, from which AB was laid off will be 50, and BC 27,05.

CASE IV.

The two legs AB and BC given, to find the angles and the hypothenuſe.

Let the leg AB be 42,05, and BC 27,05, required the angles BAC, ACB, and the hypothenuſe AC.

Firſt for the angle BAC, make BA the radius.

As BA 42,05,
: radius tangent of 45°;
:: the leg BC 27,05,
: tangent of its oppoſite angle BAC 32° 45′.

Or, for BCA, make BC the radius.

As the leg BC 27,05,
: radius tangent of 45°;
:: the leg AB 42,05,
: to the tangent of its oppoſite angle BCA 57° 15′.

By

By the logarithmic lines.

Extend the compaſſes from the firſt term 42,05 on the line of numbers, to the third term 27,05 on the ſame line; that extent will reach from 45° at the end of the line of tangents to 32° 45′ on the ſame line, for the angle BAC, which taken from 90°, leaves 57° 15′ for the angle BCA.

Or, if BCA be found firſt, extend from the firſt term 27,05, to the third term 42,05 on the line of numbers; that extent will reach from the end of the line of tangents, namely 45, to the ſame point in that line as before, which muſt not now be called 32° 45′, but the complement thereof, or what it is ſhort of 90°, that is, 57° 15′; for, if the firſt term in the proportion be greater then the ſecond, the third term will likewiſe be greater than the fourth, and *vice verſa.* Now, in the firſt proportion of this preſent caſe, the firſt number 42,05 being greater than the third 27,05 (which is the ſecond in working on the logarithmic lines); the ſecond term 45 (the third in working), will be greater than the fourth 32° 45′; but in the latter proportion, where the ſecond term in working on the ſcale is greater than the firſt, the fourth will be greater than the third 45°, conſequently could not be 32,45, though the compaſſes reſted upon that point; it muſt therefore be the deficiency of 32° 45′, to 90°. Indeed, on moſt ſcales the line of tangents is numbered from 1 to 45, and back again from 45 to 80, the points 50 and 40, 60 and 30, 70 and 20, &c. being placed together; then in this caſe the fourth term, when greater than 45, is ſeen by inſpection, without the trouble of taking its complement to 90°.

By the ſectoral lines.

Make the parallel diſtance of 45 and 45 on the line of tangents, equal to the lateral diſtance of 42,05 on the line of lines; then the lateral diſtance of 27,05 on the line of lines, taken between the points of the compaſſes, and applied parallelwiſe to the line of tangents, ſo that both points reſt upon the ſame number on each leg of the ſector, will give for the fourth term, 32° 45′.

And in the ſecond proportion, as the line of tangents is continued no farther than 45°, and the fourth term required will be more than that number, let the parallel diſtance of 45 and 45 on the ſmall tangents, be made equal to the lateral diſtance of the firſt term 27,05 on the line of lines, then the lateral diſtance of the third term 42,05 on the line of lines, will be equal to the parallel diſtance of 57° 15′ on the line of ſmall tangents.

The hypothenuſe may now be found by Caſe III. the angles and legs being known.

Geometrical conſtruction.

Draw the right line BA, and make it equal to 42,05; at B erect the perpendicular BC, which make equal to 27,05; from C to A draw the line CA, and the triangle is conſtructed: then will BAC, meaſured by a protractor, or the line of chords, be found equal to 32° 45′, and ACB 57° 15′; and the hypothenuſe AC, meaſured on the ſame ſcale with the two legs, will be 50.

Oblique-angled Plane Trigonometry, by the Sector and Gunter's Scale.

In the oblique triangle ABC, the longeſt ſide AB is called the baſe, and the other two, AC and BC, are called ſides of the triangle. Fig. 21, pl. 4.

In all plane triangles the ſides are proportional to the ſines of their oppoſite angles; therefore, as any one ſide is to the ſine of the angle oppoſite to that ſide, ſo is any other ſide to the ſine of its oppoſite angle, and *vice verſa.*

When an angle is to be found, the firſt term in the proportion muſt be a ſide; but to find a ſide begin with an angle.

If the angle be obtuſe, or more than 90 degrees, take it from 180 degrees, and work with the remainder;

mainder; this remainder is called the ſupplement of the angle.

CASE I.

Two angles and a ſide given, to find the other two ſides.

Let the angle ABC be 40°, BAC 30°, and the baſe AB 60 yards; required the ſides AC and BC.

For the ſide AC.

Firſt, Find the angle oppoſite to the given ſide thus:

The angle ABC 40° added to the angle BAC 30° makes 70°, which ſubtracted from 180°, leaves 110° for the remaining angle ACB, whoſe ſupplement is 70°.

Then, as ſine of ſupplement angle ACB 70°,
: oppoſite ſide AB 60;
:: ſine angle ABC 40°,
: oppoſite ſide AC 41,04 yards.

For the ſide BC.

As ſine ſupplement angle ACB 70°,
: oppoſite ſide AB 60;
:: ſine angle BAC 30°,
: oppoſite ſide, BC 31,93 yards.

By

By the logarithmic lines.

Firſt, for the ſide AC.—Extend the compaſſes from 70° to 40° on the line of ſines; that extent applied to 60 on the line of numbers will reach to 41,04 on the ſame line, for the ſide AC.

Secondly, for the ſide BC.—Extend on the line of ſines from the firſt term 70° to the third term 30°, which extent will reach from 60 on the line of numbers, to 31,93 on the ſame line, for the fourth term, or ſide BC.

By the ſectoral lines.

Firſt, to find the ſide AC.—Make the lateral diſtance of the ſecond term 60 on the line of lines, a parallel diſtance of the firſt term 70° on the line of ſines; then the parallel diſtance between 40° and 40° (the third term) on the line of ſines, will extend from the centre of the inſtrument along the line of lines, to 41,04, the fourth term, or ſide AC.

Secondly, for the ſide BC.—The firſt and ſecond terms in this proportion being the ſame as before, without altering the opening of the ſector, take the parallel diſtance of 30° and 30° on the line of ſines, which meaſured from the centre along the line of lines, will give 31,93 for the fourth term, or ſide BC.

Geometrical conſtruction.

Draw a right line as AB, and make it equal to 60 from any ſcale of equal parts; at the extremity A, with a protractor or line of chords, lay off an angle equal to 30°, and draw the line AC of an unlimited length; at the extremity B make an angle equal to 40°, and draw the line BC, which will meet the former line AC in C; ſo will ACB be the triangle required, and AC, meaſured upon the ſcale of equal parts from which AB was taken, will be 41,04, and BC 31,93 yards.

CASE II.

Two ſides and an angle oppoſite to one of them given, to find the other angles and the third ſide.

Let the ſide AB 60, AC 41,04, and the angle ABC 40° be given; required the angle ACB obtuſe, and the ſide BC.

Note. If the given angle be obtuſe, the required angle will be acute; but when the given angle is acute, and oppoſite to one of the leſs given ſides, then the required angle may be either acute or obtuſe; it ought therefore to be determined which of the two it muſt be previous to the operation: if acute, it is found at once; if obtuſe, the ſupplement of the fourth term muſt be taken to 180 degrees.

For

For the angle ACB.

As ſide AC 41,04,
: ſine of its oppoſite angle CAB 40°;
:: ſide AB 60,
: ſine of its oppoſite angle ACB 70°.

But, as by the queſtion it is required to be obtuſe, its ſupplement to 180 degrees muſt be taken, namely 110°.

Then 110° added to 40°, makes 150°; this taken from 180° leaves 30° for the third angle BAC.

For the ſide BC.

As ſine angle ABC 40°,
: oppoſite ſide AC 41,04;
:: ſine angle CAB 30°,
: oppoſite ſide BC 31,93.

By the logarithmic lines.

Extend the compaſſes from the firſt term 41,04, to the third 60 on the line of numbers; that extent will reach from the ſecond term 40° to the ſupplement of the fourth term 70° on the line of ſines; therefore the fourth term or angle ACB will be 110°.

For the ſide BC.—Extend from 40° to 30° on the line of ſines; that extent will reach from 41,04 to 31,93 on the line of numbers.

By the ſectoral lines.

Make the lateral diſtance of the firſt term 41,04, a parallel diſtance of the ſecond term 40° on the line of ſines; then the lateral diſtance of the third term 60 on the line of lines, will be a parallel diſtance of 70° and 70° on the line of ſines, which taken from 180 (becauſe the angle is to be obtuſe), gives 110° for the required angle ACB.

For the ſide BC.—The two firſt terms in each proportion being alike, let the ſector remain at the ſame opening, and take the parallel diſtance of 30° and 30° on the line of ſines; this applied laterally to the line of lines, will give 31,93 for the ſide BC.

Geometrical conſtruction.

From any ſcale of equal parts make a right line, as AB, equal to 60; at the point B lay down the given angle ABC 40°, and draw at pleaſure the line BC; from the ſame ſcale of equal parts take the given ſide AC 41,04, and with one foot of the compaſſes in the point A, croſs the line BC with the other in the point C; draw AC, and the triangle is conſtructed. Then will the angle ACB meaſure 110°, and the ſide BC 31,93.

CASE III.

Two ſides and their contained angle given, to find the other angles and the third ſide.

Let the ſide AB be 60, AC 41,04, and the angle CAB, contained between the two given ſides 30°; required the other angles and the third ſide BC.

The proportion in this caſe will be,

As the ſum of the two ſides containing the given angle, is to the difference of thoſe two ſides, ſo is the tangent of half the ſum of the angles oppoſite to theſe two ſides, to the tangent of half their difference.

This half difference, added to half the ſum of the angles, will give the greater; and ſubtracted from the half ſum, will give the leſs angle.

For the demonſtration of this proportion ſee Simſon's, Robertſon's, or any other treatiſe on trigonometry.

For

For the angles.

As ſum of AB and AC 101,04,
: their difference 18,96;
:: tangent of half ſum of the angles ACB and ABC 75°
: tangent of half their difference 35°
half ſum 75

The half ſum of the two angles is thus found.

The three angles are	180
BAC given, is	30
the two remaining are	150
half of which is	75

their ſum 110 for the greater angle ACB.
their difference 40 for the leſs angle ABC.

The greater angle is always oppoſite to the greater ſide, and the leſs angle to the leſs; therefore they are eaſily diſtinguiſhed.

The angles and two ſides being known, the third ſide BC may be found by either of the preceding caſes.

By the logarithmic lines.

Extend the compaſſes from the firſt term 101,04 to the ſecond 18,96 on the line of numbers; with this extent place one foot of the compaſſes upon 45° at the end of the line of tangents, and wherever the other falls on the ſame line, there let it reſt; then bring the leg from 45° to the third term or half ſum 75°; take up the compaſſes with this extent, and applying one foot upon 45° at the end of the line of tangents, the other will reach to the fourth term, namely 35° on the ſame line, which is the half difference.

This proportion cannot be performed by the ſectoral lines on the common ſector, the large tangents proceeding no farther than 45°, and the ſmall ones beginning at 45°.

Geometrical conſtruction.

Draw the right line AB, and make it equal to 60; at A make the angle BAC equal to 30°, and draw the line AC, on which, from the ſame ſcale of equal parts, lay off 41,04 from A to C; draw CB, and the triangle is conſtructed.

CASE IV.

The three ſides given to find the three angles.

Let the ſide AB be 60, AC 41,04, and BC 31,93; required the three angles.

The proportion in this caſe will be,

As the baſe or longeſt ſide AB 60, is to the ſum of the two ſhorter; ſo is the difference of the two ſhorter to the difference of the ſegments of the baſe, made by a perpendicular let fall from the angle oppoſite to the baſe. Then, half the difference of theſe ſegments added to half their ſum, will give the greater ſegment; and ſubtracted from half their ſum, will give the leſs ſegment.

The triangle being now divided into two right-angled triangles, one leg and the hypothenuſe in each given, the angles may be found by Caſe II. of plane trigonometry right-angled.

For a demonſtration of the above proportion, ſee Simſon's Trigonometry.

For the two ſegments AD and BD.

As the baſe AB 60,
: ſum of AC and BC 72,97;
:: difference of AC and BC 9,11,
: difference of the ſegments of the baſe 10,96

half of which is 5,48
half ſum of the ſegments, or the baſe } 30 -

ſum is 35,48 greater ſegment AD.

difference is 24,52 leſs ſegment BD.

Then for the angle CAD.

As hypothenuſe AC 41,04,
: radius ſine 90°;
:: the leg AD 35,48,
: oppoſite angle ACD 60°, which taken from 90°, leaves the angle CAD 30°.

For the angle CBD.

As hypothenuſe BC 31,93;
: radius ſine 90°;
:: leg BD 24,52;
: ſine of oppoſite angle BCD 50°, which ſubtracted from 90°, leaves 40° for the angle CBD; and the two parts ACD 60°, and BCD 50° put together, make 110° for the third angle of the triangle ACB.

By the logarithmic lines.

Extend the compaſſes from the firſt term 60 to the ſecond term 72,97 on the line of numbers, that extent will reach from the third term 9,11 to the fourth 10,96 on the line of numbers, from which the two ſegments are found as above. Then,

For the angle CAD.—Extend from the firſt term 41,04 to the third term 35,48; that extent will reach from 90° on the line of ſines, to 60° on

the

the ſame line, which taken from 90° leaves 30° equal to the angle CAD.

For the angle CBD.—Extend from 31,93 to 24,52 on the line of numbers; that extent will reach from 90° to 50° on the line of ſines, which ſubducted from 90°, leaves the angle CBD equal to 40°.

By the ſectoral lines.

Make the lateral diſtance of the ſecond term 72,97 equal to the parallel diſtance of the firſt term 60 on the line of lines; then the parallel diſtance of 9,11 meaſured along the line of lines, will give 10,96 for the fourth term, from which the two ſegments AD and BD are found. Then,

For the angle CAD.—Make the parallel diſtance of 90° and 90° on the line of ſines, equal to the lateral diſtance of 41,04 on the line of lines; then the lateral diſtance of 35,48 on the line of lines, will be the parallel diſtance of 60° on the line of ſines; this taken from 90° leaves 30° for the required angle CAD.

For the angle CBD.—Make the parallel diſtance of 90° and 90° on the line of ſines, equal to the lateral diſtance of 31,93 on the line of lines; then the lateral diſtance of 24,52 will be the parallel diſtance of 50° on the line of ſines, which ſubtracted from 90° leaves 40° for the angle CBD.

Geometrical conſtructions.

Draw a right line AB, and make it equal to 60; from the ſame ſcale take AC in the compaſſes, namely 41,04, and with one foot in A deſcribe an arc towards C; take BC 31,93 in the compaſſes, and with one foot in B croſs the former arc with the other at C; from C to each extremity of the baſe, draw CA and CB, and the triangle is conſtructed. Then if a perpendicular be let fall from the angle ACB upon the baſe AB, the part AD will be found to meaſure 35,48, and BD 24,52, as has been determined by the inſtruments. Likewiſe the angle ABC will be 40°, BAC 30°, and ACB 110°.

Application of Trigonometry to the Meaſurement of Heights and inacceſſible Diſtances.

EXAMPLE I.

At the diſtance AB, 60 yards from the foot of a building, ſuppoſe the angle of its altitude (taken with a quadrant, or a ſextant) was found to be 41°, the height of the eye being 6 feet above the level of the baſe of the building; required its height in feet. Fig. 22, pl. 4.

By

By Caſe III. of right-angled trigonometry.

As the complement of 41°, viz. 49°,
: leg AB 60 yards;
:: angle of altitude 41°,
: height in yards 52,5 yards, which multiplied by 3, gives 157,5 feet; to this add the height of the eye 6 feet, and the ſum will be 163,5 feet for the height of the building.

EXAMPLE II.

In order to obtain the diſtance of the object C, from the two ſtations A and B, the horizontal angle CAB was 60°, CBA 80°, and the diſtance between the ſtations A and B 750 yards; required the diſtances AC and BC. Fig. 23, pl. 4.

By Caſe I. of oblique-angled trigonometry.

The ſum of 80° and 60° is 140°, which taken from 180° leaves 40° the angle ACB. Then,
As ſine of angle ACB 40°,
: oppoſite ſide AB 750 yards;
:: CAB 60°,
: oppoſite ſide BC 1010, the diſtance from B. And,

As ſine angle ACB 40°,
: oppoſite ſide AB 750 yards;
:: ſine angle CBA 80°,
: oppoſite ſide AC 1150 yards, the diſtance of the object from the point A.

EXAMPLE III.

At a certain place A, two light-houſes may be ſeen, ſtanding one on each ſide of a bay; the meaſured diſtance from A to the light-houſe C was 4 miles, and the diſtance from A to the light-houſe B 6 miles, and the horizontal angle CAB, taken with a ſextant, was 31° 30′; required the width of the bay, or the diſtance from one light-houſe to the other. Fig. 24, pl. 4,

By Caſe III. of oblique-angled trigonometry.

As ſum of AC and AB 10 miles,
: the difference of AC and AB 2 miles;
:: tangent of half the ſum of ACB and ABC 74° 15′,
: tangent of half their difference 35° 20′
half ſum 74 15

	°	′
	180°	
CAB	31	30
	2)148	30
half ſum	74	15

their ſum -	109	35	for the greater angle.
their difference	38	55	for the leſs angle.

Then,
As ſine angle ABC 38° 55′,
: ſide AC 4 miles;
:: ſine angle CAB 31° 30′,
: ſide BC 3,3 miles, the diſtance of the two light-houſes.

EXAMPLE IV.

Wiſhing to obtain the perpendicular height of a mountain, I took the angle of its altitude at the foot thereof, which I found to be 61° 30′, and meaſuring from thence in a ſtraight horizontal line 400 yards, the angle of its altitude was then 36° 30′; required the height of the mountain. Fig. 25, pl. 4.

Draw at pleaſure the line AD, and ſet off thereon 400 from A to B; at A make an angle of 36° 30′, and draw AC at pleaſure; at B make an angle of 61° 30′, and draw BC, which will meet AC in C; from C let fall the perpendicular CD; then will CD be the height of the mountain.

Note. When a ſtraight line CB meets another ſtraight line AD, the two angles ABC and CBD, are equal to two right angles, or 180° (Euclid i. 13); therefore CBD being 61° 30′, the angle ABC will be 118° 30′; and this added to CAB 36° 30′, makes 155° 0′, which ſubtracted from 180, leaves 25° 0′ for the angle ACB.

Then

Then for the ſide BC.

As ſine of angle ACB 25°,
: its oppoſite ſide AB 400 yards;
:: ſine of angle CAB 36° 30′,
: its oppoſite ſide BC 562 yards, the hypothenuſe of the triangle BCD.

For the height CD.

As radius ſine 90°,
: hypothenuſe BC 562 yards;
:: ſine angle CBD 61° 30′,
: its oppoſite ſide CD 494 yards, the height of the mountain, as was required.

EXAMPLE V.

To find the diſtance between two objects that were inacceſſible, the ſtraight line PQ was meaſured 120 chains; and the angles at the end P, were APQ 85°, BPQ 40°; at the end Q were AQP 36°, BQP 75°: hence the diſtance of the objects A and B may be found. Fig. 26, pl. 4.

CONSTRUCTION.

Draw a right line PQ, and make it equal to 120; at the point P make the angle APQ 85°, and BPQ

40°;

40°, and draw at pleaſure the lines PA and PB; at the point Q, make the angle BQP equal to 75°, and AQP to 36°, and draw the lines QA and QB; then will QA meet the former line PA in the point A, and QB will meet PB in the point B; draw a right line from A to B, ſo will the points A and B repreſent the two objects, and the line AB the diſtance from one to the other.

By the logarithmic or ſectoral lines.

Firſt find the neceſſary angles, thus:

In the triangle BPQ.		In the triangle PAQ.	
The angle BPQ is	40°	The angle APQ is	85°
BQP	75	AQP	36
their ſum -	115	their ſum -	121
ſubtract from	180	ſubtract from	180
remainder is PBQ	65	remainder is PAQ	59

Then for the ſide PB.

As ſine angle PBQ 65°,
: oppoſite ſide PQ 120 chains;
:: ſine angle PQB 75°,
: its oppoſite ſide PB 127 chains.

For the ſide PA.

As ſine angle PAQ 59°,
: oppoſite ſide PQ 120 chains;
:: ſine angle AQP 36°,
: its oppoſite ſide PA 82,4.

Then in the triangle APB are known the ſides AP and PB, together with their included angle APB 45° (found by ſubtracting 40° from 85°), to find the third ſide AB, the required diſtance.

For the angles.

As ſum of AP and PB 209,4
: their difference - 44,6
: tangent of half ſum of PAB and PBA 67° 30′
: tangent of half their difference 27° 13′

Three angles	180°
APB -	45
ſum -	135
half	67° 30′

half their difference	27° 13′
half ſum -	67 30
their ſum -	94 43 is the angle PAB.
their difference	40 17 is the angle PBA.

For the ſide AB.

As ſine of angle ABP 40° 17′,
: ſide PA 82,4;
:: ſine angle APB 45°,
: ſide AB 90,1 chains, the diſtance of the two objects.

The Uſe of the Line of Rhumbs, the Logarithmic Lines of Sine Rhumbs, and Tangent Rhumbs on Gunter's Scale, explained by a few Problems in Navigation.

The principal part of navigation is performed by right-angled plane trigonometry, in which the hypothenuſe is the rhumb line on which the ſhip ſails, and is called the diſtance; the perpendicular is called the difference of latitude, and the baſe the departure from the meridian; the angle oppoſite the departure is the courſe, or angle that the ſhip makes with the meridian, and the angle oppoſite the perpendicular, is the complement of the courſe, which two angles together always make eight points or rhumbs, or 90 degrees.

In the triangle NSE, the hypothenuſe SE is the diſtance; the leg SN, the difference of latitude; the leg NE, the departure; the angle NSE, the courſe; and the angle NES, the complement of the courſe: any two of which being given (except the two laſt), the reſt may be found. Fig. 27, pl. 3.

PROBLEM I.

Courſe and diſtance given, to find the difference of latitude and departure.

EXAMPLE.

EXAMPLE.

From a port in 45° 15′ ſouth latitude, a ſhip ſails NE. by E. $\frac{3}{4}$E. 456 miles; what difference of latitde and departure has ſhe made, and what latitude is ſhe then in?

For the departure NE.

As radius ſine 90°, or 8 points,
: diſtance SE. 456 miles;
:: ſine courſe NSE. 5$\frac{3}{4}$ points (See mariner's compaſs),
: departure NE. 412 miles.

For the difference of latitude NS.

As radius ſine 90°, or 8 points,
: diſtance SE. 456 miles;
:: ſine com. courſe 2$\frac{1}{4}$ points,
: difference of latitude NS. 195 miles.

Divide the difference of latitude, 195 miles by 60 (the number of miles in a degree), and the quotient is 3° 15′; therefore, as the ſhip ſailed northerly from a place in ſouth latitude, it is evident her preſent latitude is leſs than the latitude left by the difference of latitude ſhe has made, namely, 3° 15′, and her preſent latitude is 42° 0′ S.; likewiſe the meridian ſhe is now upon, is 412 miles to the eaſtward of that ſhe departed from.

By the Logarithmic Lines on Gunter's Scale.

On the line of ſine rhumbs marked SR, extend the compaſſes from 8 at the end of the line, to the courſe 5¾, that extent will reach from the diſtance 456 miles on the line of numbers, to the departure 412 miles, on the ſame line.

2dly, Extend from 8 points to the complement of the courſe on the line of ſine rhumbs, namely, to 2¼; that extent will reach from the diſtance 456 miles, to the difference of latitude 195 miles, on the line of numbers.

CONSTRUCTION.

In the conſtruction of figures in navigation, let the upper part of the book or paper repreſent the north, and the lower part the ſouth; then the right hand ſide of the meridian will be the eaſt, and the left weſt.

Draw the meridian NS, and make S the place from which the ſhip ſailed; then with one foot of the compaſſes in S, deſcribe the chord of 60° to the right hand ſide of the meridian (the ſhip's courſe being eaſterly), and from the line of rhumbs, (beneath the line of chords), take $5\frac{3}{4}$ in the compaſſes, and apply that extent upon the arc deſcribed with the chord of 60°: draw SE, and make it equal to the given diſtance 456 miles, and from E let fall the perpendicular NE, and the figure is conſtructed. Then will NS, the difference of latitude, meaſure 195, and NE, the departure, 412 miles.

PROBLEM II.

Courſe and difference of latitude given, to find the diſtance and departure.

A ſhip ſails NE. by E. $\frac{3}{4}$ E. from a port in 45° 15′ ſouth latitude, till, by obſervation, ſhe is found to be in 42° 0′ S.; what diſtance has ſhe run, and how much departure from her firſt meridian, has ſhe made?

The difference of latitude between 45° 15′ S. and 42° 0′ S. is 3° 15′, or 195 miles; therefore,

For the diſtance SE.

As ſine complement courſe NES. $2\frac{1}{4}$ points,
: difference of latitude NS. 195 miles;
:: radius ſine 90°, or 8 points,
: diſtance SE. 456 miles.

For

For the departure NE.

As radius ſine 90°, or 8 points,
: diſtance SE. 456 miles;
:: ſine coure 5¾ points,
: departure NE. 412 miles.

By the logarithmic lines.

Extend from 2¼ the complement of the courſe, to 8 on the line of ſine rhumbs; that extent will reach from the difference of latitude 195 miles, to the diſtance 456 miles on the line of numbers.

2dly. Extend from 8 points to the courſe 5¾ on the line of ſine rhumbs; that extent will reach from the diſtance 456 to the departure 412 miles.

CONSTRUCTION.

Draw the meridian line NS, and make it equal to the given difference of latitude 195 miles; at the point N erect the perpendicular NE of an unlimited length; at the point S make an angle with the chord of 60°, and the line of rhumbs, as above directed, equal to 5¾ points, and draw the line SE, which will meet NE in the point E; NSE will be the triangle, in which NE, the departure, will meaſure 412 miles, and SE, the diſtance, 456 miles.

PROBLEM III.

Difference of latitude and departure given, to find the courſe and diſtance.

A ſhip ſails in the NE. quarter, from a port in 45° 15′ S. till by an obſervation of the ſun ſhe is in 42° 0′ S. at the ſame time ſhe has departed 412 miles from her firſt meridian; required the courſe ſhe has ſailed upon, and the diſtance run.

The difference of latitude, as found in the preceeding problem, is 195 miles.

For the courſe, by Caſe IV. of right-angled trigonometry.

As difference of latitude NS. 195 miles,
: radius tangent 45°, or tangent 4 points;
:: departure NE. 412 miles,
: tangent courſe NSE. $5\frac{3}{4}$.

For the diſtance SE.

As ſine courſe $5\frac{3}{4}$,
: departure 412 miles;
:: Radius ſine 90°, or 8 points,
: diſtance SE. 456 miles.

By the logarithmic lines.

Extend the compaſſes from 195 to 412 on the line of numbers; that extent applied upon the radius on the line of tangent rhumbs, namely, 4 points (the tangent of 4 points, or 45 degrees, being equal to the ſine of 8 points, or 90 degrees), will reach to $2\frac{1}{4}$ on the ſaid line, which in this caſe muſt be taken from 8 points; for as the ſecond term is greater then the firſt, the fourth muſt be greater than the third; therefore the courſe is 8 points leſſened by $2\frac{1}{4}$, or $5\frac{3}{4}$ points.

2ndly. Extend the compaſſes from $5\frac{3}{4}$ to 8 on the line of ſine rhumbs; that extent will reach from the departure 412 miles, to the diſtance 456 miles on the line of numbers.

CONSTRUCTION.

Draw the meridian line NS, and make it equal to 195 miles; at the point N erect the perpendicular NE, equal to 412 miles; draw SE, and it is done; then will the angle NSE, meaſured by the line of rhumbs and chord of 60°, be $5\frac{3}{4}$ points, or NE. by E. $\frac{3}{4}$E. for the courſe, and SE. on the ſcale of equal parts will be 456 miles for the diſtance.

Of

Of the additional Inſtruments generally found in a Magazine Caſe.

Theſe are,
A lengthening piece for the large compaſſes.
A pair of hair compaſſes.
A pair of proportional compaſſes.
A pair of triangular compaſſes.
A pair of gunner's calipers.

Of the lengthening piece.

When any extent is to be taken, or circle deſcribed whoſe radius is too large for the opening of the common compaſſes, then this piece is applied to one leg of the large compaſſes to increaſe their extent. It is repreſented at Fig. G. pl. 1.

Of the hair compaſſes.

Theſe are a pair of very neatly made compaſſes with two ſteel points, one of which contains a ſpring and ſcrew, and by theſe diſtances may be laid off, or meaſured to the niceſt accuracy. The manner of uſing them is this: open the compaſſes nearly to the required extent, then turn the ſcrew, and the moveable point may be extended or cloſed the leaſt imaginable, or to a hair's breadth, as it were, for which reaſon they are called hair compaſſes. Fig. H, pl. 1.—Fig. I, pl. 1, is a pair of ſmall compaſſes for ſtriking circles on copper.

Of the proportional compaſſes.

This inſtrument conſiſts of two flat pieces of braſs, laid one upon the other ſo as to appear one piece when ſhut. Theſe pieces ſlide over one another, moving round a center, which is likewiſe moveable itſelf, along a groove or channel cut through the compaſſes, nearly the whole length. To this center is fixed a ſliding pair of braſs, with a fine hair line drawn thereon, which ſerves as an index to be placed againſt the diviſions marked on each ſide of the groove on the compaſſes.

The divided lines are four.

1. A line of lines.
2. A line of ſuperficies, or areas.
3. A line of ſolids.
4. A line of circles or polygons inſcribed in circles. Fig. K, pl. 1, repreſents a pair of theſe compaſſes.

The Uſe of the Line of Lines.

This line is marked with 10 unequal diviſions, numbered from 1 to 10; its uſe is to divide lines into any required number of equal parts, or in any proportion, &c.

PROBLEM I.

To divide a given line into two equal parts, or to biſect it.

Move the centre along the groove till the index ſtands againſt 2 (the number of parts into which the given line is to be divided), on the line of lines, and there ſcrew it faſt. Then open the compaſſes, and between the points of the longer legs, from the centre, take the extent of the given line, and the diſtance between the points of the ſhorter legs will be one half of the diſtance between the longer, conſequently one half of the given line.

PROBLEM II.

To divide a given line into any number of equal parts.

Let a line of 5 inches be given, to be divided into 3 equal parts.

Move the centre along the groove till the index points at 3, on the line of lines; then from any ſcale of equal parts, take the extent of 5 between the points of the longer legs of the compaſſes, then the diſtance between the points of the ſhorter will be one third of the diſtance between the points of the longer, conſequently, if a line of 5 from any ſcale of equal parts be drawn, and the extent of the ſhorter legs be applied thereupon, it will divide the line into 3 equal parts, which was required to be done. Thus may a line of any other length be divided into any number of equal parts from 2 to 10.

The Uſe of the Line of Circles.

By the line of circles any regular polygon whoſe ſides do not exceed the numbers on the ſcale, may be inſcribed in a given circle.

PROBLEM.

To inſcribe in a given circle a nonagon, or figure with 9 ſides.

Let the radius of the given circle be 3 inches; it is required to inſcribe a nonagon therein.

Shut the compaſſes, and unſcrew the nut; then ſlide the centre along the groove till the index coincides with the 9th diviſion on the ſcale of circles, and there ſcrew it faſt. Open the compaſſes and take the extent of the given radius 3 inches, between the points of the longer legs; then will the diſtance between the points of the ſhorter be a ſide of the required polygon.

PROBLEM.

Having the ſide of any regular polygon given, to conſtruct the ſame.

Let it be required to conſtruct a decagon, or figure with 10 equal ſides, each ſide being 2 inches.

This is evidently the reverſe of the laſt problem: therefore ſhut the compaſſes, and unſcrew the nut; then ſlide the center till the index points to the 10th diviſion upon the ſcale of circles, and ſcrew it faſt. Open the compaſſes, and take the given ſide between the points of the ſhorter legs, and the diſtance between the points of the longer will be the length of the radius of a circle, in which the decagon will be inſcribed.

If the index be ſet to the 6th diviſion on the line of circles, and the compaſſes opened to any extent, the diſtance between the points of both legs will always be the ſame, becauſe the radius of a circle is equal to a ſide of an inſcribed hexagon.

The Uſe of the Line of Planes, Superficies, or Areas.

The line of planes, generally marked Plans upon the inſtrument, is intended to ſhew the proportion of plane or ſuperficial figures to each other,

whoſe

whoſe ſides, diameters, &c. are known, as alſo to point out the roots of ſquare numbers from 1 to 10, from 100 to 1000, &c. The numbers on this line proceed from 1 to 10 by unequal diviſions, and are ſo laid down, that if the points of the longer legs be opened to any ſquare number (the index on the ſliding center at the ſame time being ſet to that number), the diſtance between the points of the ſhorter, applied to the ſame ſcale from which the diſtance between the longer was taken, ſhall be equal to the ſquare root of that number.

1ſt. To find the ſquare root of a given number.

Let the ſquare root of 4 be found by the proportional compaſſes.

Shut the compaſſes, and unſcrew the nut; ſlide the center along the groove till the index points to the number 4 upon the line of plans, and there ſcrew it faſt. Open the compaſſes, and from any ſcale of equal parts on the plane ſcale, or elſewhere, take 4 between the points of the longer legs; apply the points of the ſhorter legs upon the ſame ſcale, and the diſtance between them will be equal to the ſquare root of the diſtance between the points of the longer legs, which in this caſe is 2, the ſquare root of the given number 4.

In like manner the ſquare root of

400 is 20,
40000 is 200,
400000 is 2000, &c.

Likewiſe, if the index be ſet to 9, the root of 9 will be 3, or the diſtance between the ſhorter points of the compaſſes will be $\frac{1}{3}$ of the diſtance between the longer.

And the ſquare root of

900 is 30,
90000 is 300,
900000 is 3000, &c.

2dly. To find a mean proportional between two given numbers.

Required the mean proportional between 2 and $4\frac{1}{2}$.

The way to find a mean proportional between two numbers is, to multiply them together, and extract the ſquare root of the product; therefore open the compaſſes (with the index ſet againſt 9, the product of the two given numbers) till the diſtance of the larger legs be equal to 9, taken from ſome ſcale of equal parts, then the diſtance between the points of the ſhorter will be equal to 3 of thoſe parts, which is the mean proportional to 2 and 4½. For, as 2 : 3 : : 3 : 4½. Hence,

3dly. To find the ſide of a ſquare that ſhall be equal to a given parallelogram, or a ſuperficies whoſe length and breadth are given.

Required the ſide of a ſquare equal to a parallelogram, whoſe length is 36, and breadth 16 inches.

36×16=376, the area of the parallelogram. Therefore ſlide the center along the groove till the index points to a little beyond the half way between 5 and 6 on the line of planes; open the compaſſes, and from any ſcale of equal parts take 576 between the points of the longer legs; then the points of the ſmaller applied upon the ſame ſcale, will give 24 for the ſide of a ſquare that is equal in area to a parallelogram, whoſe length is 36, and breadth 16; for 24×24=576.

The Uſe of the Line of Solids.

The line of ſolids marked upon the inſtrument, Solids, and numbered from 1 to 10 with unequal diviſions, is laid down in ſuch a manner, that if the cube of any number be taken from a ſcale of equal parts between the longer points (the index at the ſame time ſtanding oppoſite to that number), the diſtance between the ſhorter will ſhew the cube root of that number.

To extract the cube root of a given number.

Let it be required to find the cube root of 8 by the line of ſolids.

Shut the compaſſes, and unſcrew the nut; move the centre along the groove till the index points to 8 on the line of ſolids, and there ſcrew it faſt; open the compaſſes, and take 8 between the longer points, from any ſcale of equal parts; then the diſtance between the ſhorter will be the cube root of the given number, which in this inſtance is 2.

The lines of plans and ſolids are now ſeldom found on the proportional compaſſes, being very ill adapted to that inſtrument on account of the ſmallneſs thereof, which throws the diviſions of the integers ſo cloſe to each other, as not to admit of their being ſubdivided. — Indeed the ſector is a much more uſeful and convenient inſtrument than the proportional compaſſes, and there can no lines or ſcales be placed upon the latter but what may, with equal propriety, be laid down upon the former. It is therefore adviſable for engineers, ſurveyors, &c. to procure the very beſt ſectors that can be made, their uſe in the mathematics, geometry, architecture, &c. being more extenſive and univerſal than that of any other inſtrument.

Of the Triangular Compaſſes. *Fig.* L, *pl.* 1.

Theſe are a pair of common compaſſes, to the head of which is fitted an additional leg by a joint and ſocket, ſo that it may be moved in almoſt every direction. Theſe compaſſes are ſerviceable in copying figures, drawings, &c. though they are ſeldom made uſe of.

PROBLEM.

To copy a right-lined figure by the triangular compaſſes. Fig 28, pl. 4.

Let it be required to draw a figure ſimilar and equal to the irregular polygon ABCDE.

OPERATION.

Take any of the ſides, as AB, between the legs of the compaſſes, and move the third leg to the point C. With this extent of the three legs make three points upon the paper which is to contain the copy, and between theſe points draw the lines AB and BC; then place two legs of the compaſſes upon the points A and C of the given figure, and extend the third to D; take up the compaſſes, and applying two of the legs upon the points A and C, already obtained, in the figure to be made, the third leg will give the point D. Draw the line CD. Place two legs of the compaſſes upon the points A and D of the given figure, and extend the third to E. Then take up the compaſſes, and ſetting two of the legs upon the points A and D of the figure to be drawn, the third will give the point E. Draw AE and DE, ſo will you have an exact copy of the given figure ABCDE. In like manner may any other right-lined figure be copied.

Of Gunners' Calipers.

A pair of calipers is an inſtrument for meaſuring the diameters of convex and concave bodies, by taking their extent between the points, as a right line is taken by a pair of common compaſſes.

A pair of Gunners' calipers is an inſtrument uſed by engineers for meaſuring the diameters of ſhot; the bore, or calibre of guns; and many other purpoſes in gunnery.

This inſtrument conſiſts of two flat plates of braſs, which are moveable quite round a joint, the plates ſliding one over the other. Upon the faces of theſe plates are a variety of lines, tables, &c. ſome of which are particularly eſſential to this inſtrument, but many of them might, with equal propriety, be laid down upon the ſector or plane ſcale.

On the Calipers made by Meſſrs. Watkins are contained the following Lines, Tables, &c.

On one face of the inſtrument, which we ſhall diſtinguiſh by the letter A, are,

1. A ſcale of inches.
2. A table ſhewing the weight of a cubic foot of various metals, ivory, water, &c. and their ſpecific gravities.
3. A table, ſhewing the quantity of powder for proof and ſervice charges of iron guns from ½ to 42 pounders.
4. A line of inches for meaſuring the diameters of the calibres of guns.
5. A line joining to the preceding, to ſhew the weight of ſhot proper for given gun-bores. This line is marked—Guns.
6. Two concentric ſemicircles, each divided into 180 degrees, but proceeding in contrary directions. The centre of theſe ſemicircles is the joint round which the legs of the calipers move.
7. A line of inches, with an index on the circular head of the calipers, for meaſuring convex diameters.

On the other face of the calipers, which we will call B, are,

8. A table ſhewing the quantity of powder for filling the chambers of braſs mortars and howitzers.

9. A table ſhewing the quantity of powder for proof and ſervice charges of braſs guns.

10. A line of lines, marked L.

11. Two circular lines on the head of the calipers, with an index marked—Shot.

12. Various mathematical figures, promiſcuouſly placed on the inſtrument, with numbers affixed to each.

1. *Of the ſcale of inches.*

This line is placed along the exterior edge of each plate of the calipers, ſo that, when they are opened out, it may form one ſtraight line; its uſe is well known.

2. *Of the table of weights and ſpecific gravities of bodies.*

This table takes up the whole face of one of the plates of the calipers, excepting the line of inches above deſcribed.—This table is not eſſentially neceſſary to be engraved on the calipers, but might be placed on any other flat inſtrument, as the ſector, plane ſcale, parallel ruler, &c. It conſiſts of three columns: the firſt contains the name of metal, &c. the ſecond the weight of a cubic foot of that metal, and the third the ſpecific gravity. It runs as follows:

A

TABLE

Shewing the Weights and Specific Gravities of Bodies.

Bodies.	Weight of a Foot.	Specific Gravities.
Fine gold - -	1352,4℔	11,365
Gold - -	1180,4	10,330
Quickſilver -	874,9	8,101
Lead - -	707,7	6,553
Fine ſilver -	693,1	6,418
Silver - -	658,3	6,096
Copper - -	502,4	5,208
Caſt braſs -	500,0	4,630
Steel - -	490,7	4,544
Iron - -	477,5	4,422
Tin - -	457,4	4,236
Marble -	169,3	1,568
Glaſs - -	161,2	——
Alabaſter -	113,0	——
Ivory - -	113,9	1,550
Sea water - -	64,3	0,594
Common water	62,3	0,578
Dry oak - -	57,8	0,536

At the end of the above Table is the following ſmall one:

A Cylindric Foot.	Weight.
Sea water - -	50,481
Common water -	49,106
A Cylindric Inch.	**Weight.**
Sea water - -	0,029
Common water -	0,028

There are few tables of the weights and ſpecific gravities of bodies that exactly agree together, ſome weighing the bodies to a nicer degree of accuracy than others; but the moſt material difference ariſes from the bodies themſelves being weighed in different ſtates. The above table is by no means ſtrictly true, though ſufficiently ſo for the purpoſe it is intended.

By the ſpecific gravity of a body, is meant the proportion which that body has to ſome other body fixed upon as a common ſtandard. Rain water is generally taken for this common ſtandard, and then the ſpecific gravity that any body is ſaid to have, is expreſſed by a number ſignifying how many times it is heavier or lighter than rain water.

The ſpecific gravities of two bodies of equal bulks are in proportion to one another as their weights; therefore, if the weight of a cubic foot of any body be divided by the weight of a cubic ſoot of water, the quotient will be the ſpecific gravity of that body, compared with rain water.

Note. The body by which the ſpecific gravity of any other body is compared, ſhould be repreſented by unity, or 10, or 100, 1000, &c. for convenience ſake; but any number, whether whole or fractional, will anſwer the ſame purpoſe. The foregoing table has 0,578 for rain water.

Uſe of the Table.

1ſt. The dimenſions or ſolidity of a body being given, to find its weight.

RULE.

Multiply the ſolidity of the body given in feet, into the number in the middle column of the table oppoſite the name of the given body, and the product will be its weight.

EXAMPLE.

Required the weight of a ſolid block of copper, whoſe length is 4 feet, breadth $2\frac{1}{2}$ feet, and depth $\frac{1}{2}$ a foot.

Multiply the three dimenſions together for the ſolidity.

4×2, 5×,5=, 5 ſolid feet.

A cubic foot of copper weighs 502,4 pounds.

Multiply 502,4 by 5, and the product is 2512 pounds, the weight required.

2dly. The weight of any body being given, to find its ſolidity.

RULE.

Divide the weight of the body by the number in the middle column of the table oppoſite the name of the body, and the quotient will be the required ſolidity of the body in feet.

EXAMPLE.

If 3 tons of caſt braſs be run into pigs 2 feet long, 6 inches wide, and 3 inches deep; how many ſuch pigs will be contained in the 3 tons?

3 multiplied by 20 gives 60 cwt.; and 112 multiplied by 60 gives 6720 pounds.

In the middle column of the table oppoſite caſt braſs, ſtands 500, the number of pounds in a cubic foot.

And 6720 divided by 500 gives 13,44, the number of cubic feet in 3 tons.

Now find the ſolidity of each pig.

Two feet, the length, multiplied into $\frac{1}{2}$ a foot, the breadth, and that product into $\frac{1}{4}$ of a foot, the depth, will give $\frac{1}{4}$ of a ſolid foot for the ſolidity of each pig; but the whole ſolidity of the metal is 13,44 feet; therefore divide 13,44 by $\frac{1}{4}$, or, which is the ſame thing, multiply it by 4, and the product is 53,76 for the number of pigs required to be found.

3. *Of the table for ſhewing the quantity of powder for iron guns.*

This table is engraved on one plate of the face A of the calipers, and conſiſts of three columns: the firſt, marked iron guns, ſhews the weight of the ſhot; the ſecond, marked proof, the quantity of powder for a proof charge; and the third, marked ſervice, ſhews the quantity of powder for a ſervice charge.

The following is the table:

Iron Guns.	Proof.		Service.	
	lb.	oz.	lb.	oz.
½	0	8	0	4
3	3	0	1	8
4	4	0	2	0
6	6	0	3	0
9	9	0	4	8
12	12	0	6	0
18	15	0	9	0
24	18	0	11	0
32	21	0	14	0
42	25	0	17	0

Uſe of this table.

The uſe of the above table is obvious; look for the nature of the gun in the firſt column, and oppoſite to it in the ſecond ſtands the quantity of powder for a proof charge, and in the third, the quantity of powder for a ſervice charge. Thus for an iron 12 pounder, the quantity of powder to prove the ſame is 12 pounds, but for common ſervice the allowance is 6 pounds. For a 42 pounder, the proof charge is 25 pounds, and the ſervice charge 17 pounds.

4. *Of the line of inches for meaſuring concave diameters.*

On the lower part of the ſame plate on which the above table ſtands, is a line marked—Inches. It begins near the ſteel point of the calipers with the number 2, and proceeds to 10. Each diviſion is ſubdivided into halves and quarters.

The uſe of this line is to find the diameter of the calibre, or bore of cannon.

Slide the legs or plates of the calipers acroſs each other until the ſteel points touch the concave ſurface of the bore of the gun at its greateſt breadth. Take out the calipers, and obſerve what diviſion on the line of inches is cut by the edge of the other plate, for that will be the calibre of the gun.

5. *Of the line ſhewing the weights of ſhot proper for given gun-bores.*

Between the laſt mentioned line of inches and the table preceding it, on the ſame leg of the calipers, there is a line marked—Guns. It begins near the ſteel point of the calipers with the number 1, proceeding with unequal diviſions in the following order, 1, $1\frac{1}{2}$, 2, 3, 4, 5, 6, 8, 9, 10, 12, 16, 18, 20, 24, 26, 32, 36, 42.

The uſe of this line is to ſhew the weight of ſhot proper for given gun-bores, from a 1 to a 42 pounder.

EXAMPLE.

Having meaſured the concave diameter of the bore of a gun by the calipers, according to the directions given in the laſt article, and found it to be 4 inches; what weight muſt a ſhot be that is proper for ſuch a piece?

The bore being taken between the points of the calipers, the line of inches will be cut in the fourth diviſion as mentioned in the queſtion, and the line marked—Guns, will at the ſame time be cut in the number 8 by the edge of the calipers, ſignifying thereby, that an 8 pound ſhot is of a proper weight for a gun whoſe calibre is 4 inches in diameter.

EXAMPLE II.

The calibre of a gun, according to the calipers, being 5¾ inches diameter, what muſt be the weight of a ſhot proper for the ſaid gun?

Set the calipers to 5¾ on the line of inches, and the line marked—Guns, will then be cut by the other leg of the calipers at the diviſion 24, which is the weight of a ſhot proper for a gun whoſe diameter is 5$\frac{3}{4}$ inches.

Note. The diameters correſponding to theſe weights, are ſomething larger than thoſe given for the ſame weights of ſhot in the next following article; the reaſon of this is an allowance generally made, called Windage.

6. *Of the line of inches on the circular head of the calipers.*

This line is engraved on the circular head of the face A, and proceeds from 0 to 10. Each diviſion is ſubdivided into halves and quarters. Upon the bevel edge, which ſlides round the circular head, is an index, marked—Inches, pointing to the circular diviſions.

The uſe of this line is to meaſure the convex diameters of ſhot, &c.

Open the points of the calipers, and take between them the greateſt diameter of the ſhot; then the index will point to a number on the circular head, expreſſing the extent between the points of the calipers, in inches and parts of an inch. A pair of 6 inch calipers will extend a diameter of 10 inches.

7. *Of the ſemicircular head divided into* 180 *degrees.*

Theſe degrees are drawn round two concentric ſemicircles, forming the upper half of the round head on the face A. The outward ſemicircle has 180 degrees numbered from right to left. The inner ſemicircle is likewiſe divided into 180 degrees, but is numbered from left to right.

The uſe of theſe degrees is to meaſure angles, find the elevation of cannon, &c.

1ſt. To meaſure an internal angle, or, as it is termed in fortification, an entering angle.

Apply the legs of the calipers, ſo that the outward edges of the ſame coincide with the legs of the given angle; then will the bevel edge cut the degrees in the outward ſemicircle that are contained in the given angle.

2dly. To meaſure an external, or ſaliant angle.

Slide the legs of the calipers over each other, and let their outſide edges be made to coincide with the legs of the given angle, then will the bevel edge cut the degrees in the inner ſemicircle, which are contained in the meaſure of the given angle.

3dly. To find the elevation of a cannon or mortar.

Put a ſtraight ſtick or rod into the bore of the cannon or mortar, and let it project beyond the mouth of the cannon; to the end of this rod, without the mouth of the cannon, tie a ſtring with a plummet at the end; then meaſure with the calipers the angle formed by the rod and the ſtring, which will be the complement of the angle of elevation, or what it wants of 90 degrees. Thus, if the bevel edge cut 30 degrees on the calipers, the angle of elevation would be 60 degrees; if 40, the angle with the horizon, would be 50 degrees, &c.

8. *Of the Table for ſhewing the quantity of Powder for Mortars and Howitzers.*

On one leg of the face B of the calipers, is a table ſhewing what quantity is neceſſary for filling the chambers of braſs mortars and howitzers, by ſea and land. It conſiſts of three columns, as follows:

Braſs Mortars.	Sea.	Land.
in.	lb. oz.	lb. oz.
13	30 0	9 0
10	12 0	3 8
8	—	2 0
5½	—	0 9½
4½	—	0 5
Braſs Howitzers.		
in.	—	—
8	—	3 8
5½	—	1 0

The uſe of the above table needs no illuſtration.

9. *Of the Table for ſhewing the Qudntity of Powder for Braſs Guns.*

On one leg of the face B is the following table, to ſhew the quantity of powder neceſſary for proof and ſervice charges of braſs guns.

Braſs Guns.	Heavy.		Mid.		Light.	
	Proof.	Service.	Proof.	Service.	Proof.	Service.
	lb. oz.	lb. oz.	lb. oz.	lb. oz.	lb. oz.	lb. oz.
½	0 8	0 4	——	——	——	——
3	3 0	1 8	3 0	1 8	1 8	0 12
6	6 0	3 0	6 0	2 8	3 0	1 8
9	9 0	4 8	——	——	——	——
12	12 0	6 0	9 0	4 8	6 0	3 0
18	18 0	9 0	——	——	——	——
24	21 12	12 0	18 0	9 0	10 0	5 0
32	26 12	16 0	——	——	——	——
42	31 0	21 0	——	——	——	——

The uſe of the above table.

Look for the nature of the gun in the firſt column, and if it be of a heavy ſize, the proof and ſervice charges will be ſeen in the columns under the title heavy; if middle ſized, under the columns marked mid. and if light, under the columns marked light.

Thus if I would know the quantity of powder for a proof and ſervice charge of a heavy 32 pounder. Oppoſite 32 ſtands 26 pounds 12 ounces for the proof charge, and 16 pounds 0 ounces for the ſervice charge, both under the title Hea.; and as there are no figures in the other columns oppoſite 32, it ſhews that middling and light 32 pounders are not in uſe.

10. *Of the Line of Lines, marked* L.

This line of lines ſerves for a ſcale of equal parts, to work proportions, divide lines, and in ſhort may be applied to the ſolution of every problem which is laid down in the deſcription and uſe of the ſector.

11. *Of the two circular Lines round the Head of the Calipers.*

Theſe two lines are laid down upon the bevel edge of the head of the calipers on the face B. They contain the following figures, $\frac{1}{2}$, 1, $1\frac{1}{2}$, 2, 3, 4, 5, 6, 8, 9, 12, 16, 18, 24, 26, 32, 36, 42, which are the weights of ſhot or balls. Upon the leg of the calipers is an index marked—Shot, which belongs to theſe ſcales of weights.

The uſe of theſe lines is to ſhew the weights of iron ſhot when the diameter is taken between the points of the calipers.

EXAMPLE.

If the diameter of a ball be taken between the points of the calipers, and found to be $5\frac{3}{4}$ inches, what is the weight of that ball?

The index marked—Inches, being ſet to $5\frac{3}{4}$, according to article 6, the index marked—Shot, will point to 26 on the circular diviſions, which is the weight in pounds of a ſhot of iron whoſe diameter is $5\frac{3}{4}$ inches.

12. *Of the Mathematical Figures placed on the Calipers.*

On different parts of the inſtrument are placed the following mathematical figures, and numbers affixed to them.

1. A circle with 2 diameters, in which are the numbers 7, 22, and 113, 355.

The uſe of theſe numbers is to find the diameter of a circle having the circumference given, or to find the circumference, having the diameter given.

For, As 7 : 22 :: the diameter : the circumference;
As 22 : 7 :: the circumference : the diameter.
Or, As 113 : 355 :: the diameter : the circumference;
As 355 : 113 :: the circumference : the diameter.

Example.

What is the circumference of a circle whoſe diameter is 20?

As 7 : 22 :: 20 : $62\frac{6}{7}$ the circumference.

2. This conſiſts of a large ſquare, in which is a circle inſcribed; in that circle is a ſquare inſcribed; and within that ſquare is a circle inſcribed. To this figure are placed the numbers 28,22, and 14,11. Theſe numbers mean, that if the area or content of the outward ſquare be 28, the area of the inſcribed circle will be 22; and that the area of a ſquare inſcribed in that circle will be 14; and the area of its inſcribed circle will be 11.

The uſe of theſe numbers is to find the area of a circle, having its diameter given.

EXAMPLE.

What is the area of a circle whoſe diameter is 20?

The area of a ſquare whoſe ſide is 20 will be $20 \times 20 = 400$.

Then, As 28 : 22 :: 400 : 314,2, the area;

Or, As 14 : 11 :: 400 : 314,2.

3. This figure repreſents a ſphere or globe inſcribed in a cube; the number $246\frac{1}{4}$ ſhews the weight in pounds of an iron ſphere of 12 inches diameter.

4. This figure repreſents a cube inſcribed in a ſphere; the number $89\frac{1}{5}$, ſignifies that a cube of iron inſcribed in a ſphere of 12 inches diameter, weighs $89\frac{1}{5}$ pounds.

5. This figure is intended to repreſent a cube of iron 12 inches each ſide, and the number 464,5, to ſhew that ſuch a cube of iron weighs 464,5.

The pyramid annexed to the cube, whoſe baſe and height are each 12 inches, will weigh one third of the cube, namely 154,8 pounds.

6. This figure repreſents a cylinder of iron whoſe diameter and height are each 12 inches. The numbers 364,5 ſhews, that a cylinder of iron whoſe baſe and height are each 1 foot, will weigh 364,5 pounds.

The cone annexed to the cylinder whoſe diameter and height are each 12 inches, will be one third of the weight of the cylinder, namely 121,5 pounds. Cones and cylinders having equal heights and baſes, being in proportion to one another, as 1 to 3.

Some uſes of the above figures.

All ſolid bodies being in proportion to one another, as the cubes of their ſides, lines, diameters, &c. and the weights of like bodies, as their ſolidities; it follows, that if the dimenſions and weight of any body be given, the weight of another body of the ſame kind may be found from its dimenſions.

EXAMPLE I.

Required the weight of a cube of iron, each ſide being 2 feet.

By the fifth figure we find, that a cube of iron whoſe ſide is 12 inches or 1 foot, weighs 464,5 pounds; therefore,

As 1, the cube of 1 foot,
: 464,5, the weight;
:: 8, the cube of 2 feet,
: 3716 pounds, the weight of a cube of iron whoſe ſide is 2 feet.

Example II.

What is the weight of an iron ſhot whoſe diameter is 6 inches?

Now 6 inches $=\frac{1}{2}$ a foot, or 5;
And ,5×,5×,5=,125;
Then, As 1, the cube of 1 foot,
: 246,25, the weight of a circular foot;
:: ,125, the cube of half a foot,
: 30,78 pounds, the weight of an iron ball whoſe diameter is 6 inches.

But the weight of an iron ſhot may be more eaſily obtained, and to a tolerable degree of accuracy, from a 9 pounder, which is found to be 9 inches in diameter: therefore,

As 64, the cube of 4 inches,
: 9 pounds, its weight;
:: 216, the cube of 6 inches,
: 30,4 pounds, the weight of a ball of 6 inches diameter, which is nearly the ſame as that found above.

Example III.

A parcel of old ſhot and cannon weighing 5 tons, is to be melted down and caſt into ſhot of 3 and 5 inches diameters, and to be a like quantity in weight of each ſort; what will be the number of each?

First, 5 tons are equal to 11200 pounds,
Half of which is 5600 pounds;
Then 3 inches = $\frac{1}{4}$ of a foot,
And the cube of $\frac{1}{4} = \frac{1}{64}$.

Then, As 1, the cube of 1 foot,
: 246,25, the weight of a ſpherical foot;
:: $\frac{1}{64}$, the cubic of $\frac{1}{4}$ of a foot,
: 3,84, = the weight of an iron ſhot whoſe diameter is 3 inches.

Then, 5 inches = $\frac{5}{12}$ of a foot,
And the cube of $\frac{5}{12} = \frac{125}{1728}$.
Therefore, As 1, the cube of 1 foot,
: 246,26, the weight of a ſpherical foot;
:: $\frac{125}{1728}$, the cube of $\frac{5}{12}$ of a foot,
: 17,81 pounds, the weight of an iron ſhot whoſe diameter is 5 inches.

Divide half the weight of the metal 5600, by 3,84, the weight of a ſhot of 3 inches diameter, and the quotient is 1458, the number of the leſs kind.

Divide half the weight of the metal 5600 by 17,81 pounds, the weight of a ſhot whoſe diameter is 5 inches, and the quotient is 314, the number of ſhot whoſe diameters are 5 inches.

THE END.

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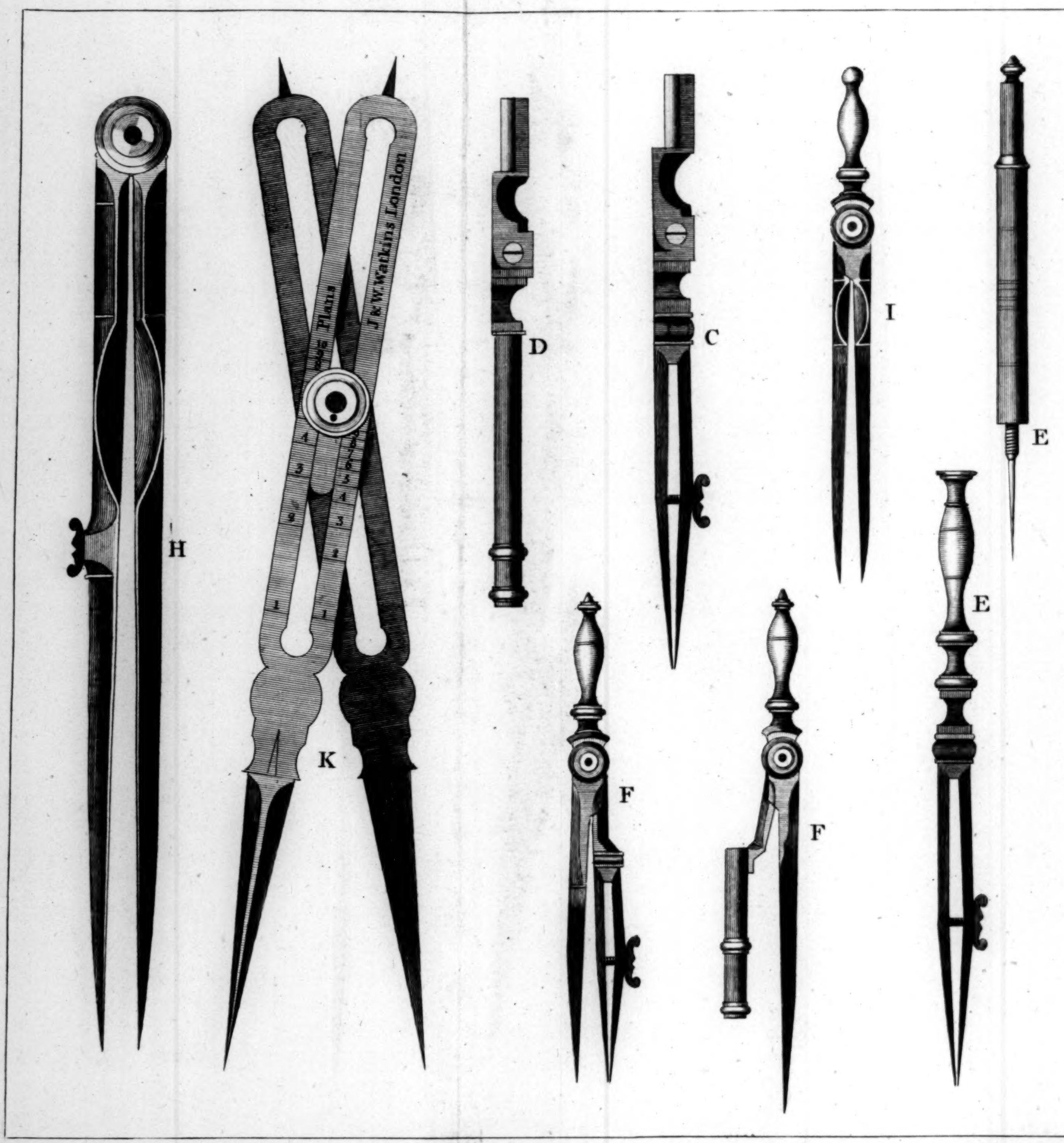

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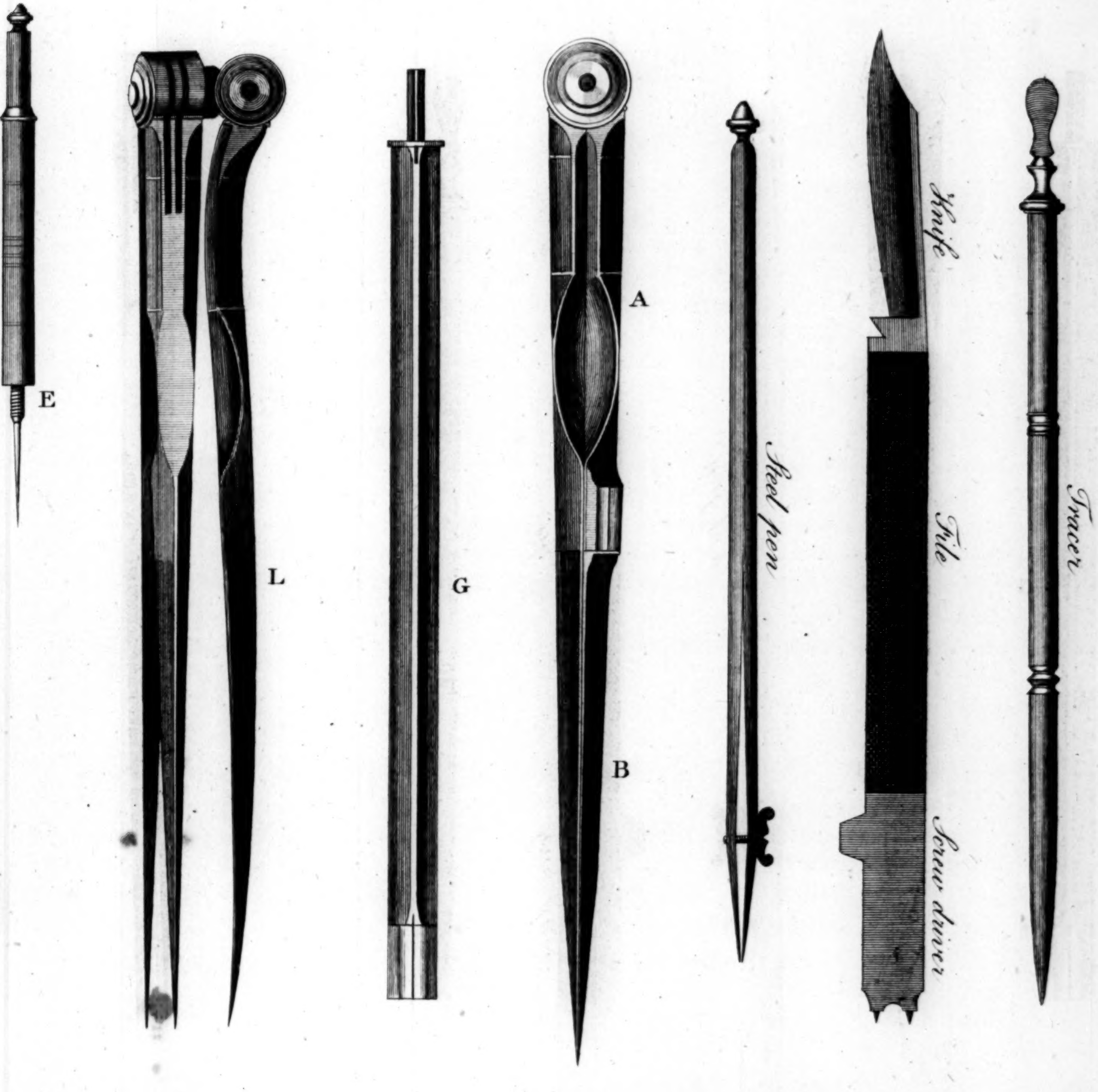
Plate I.



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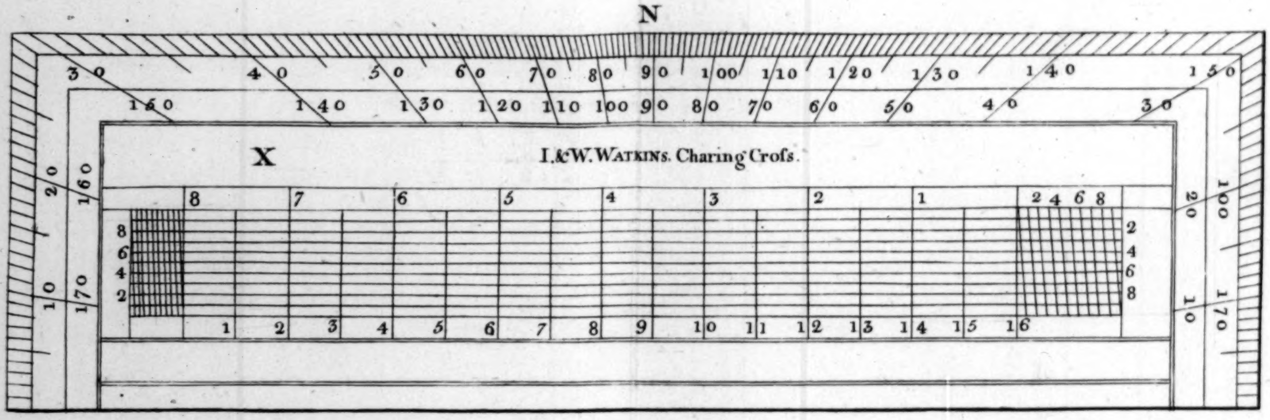

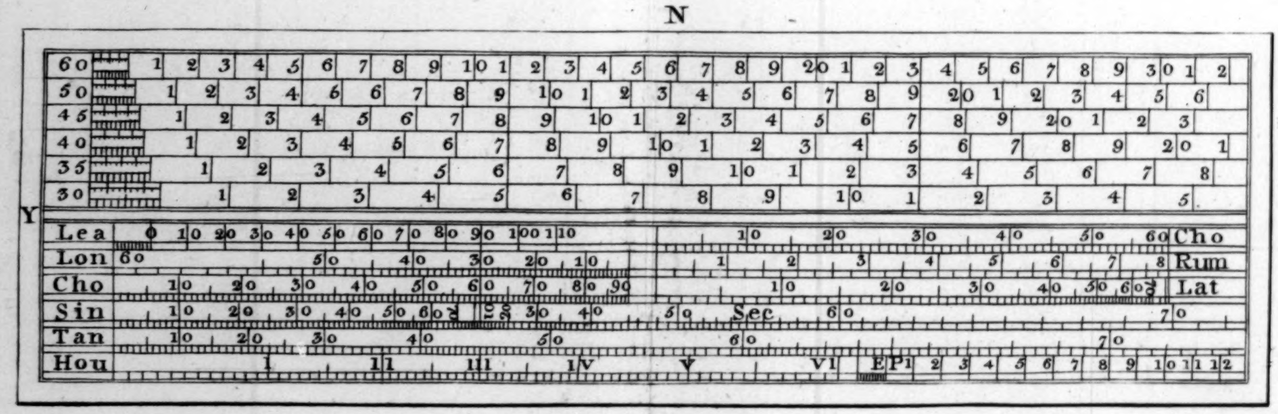

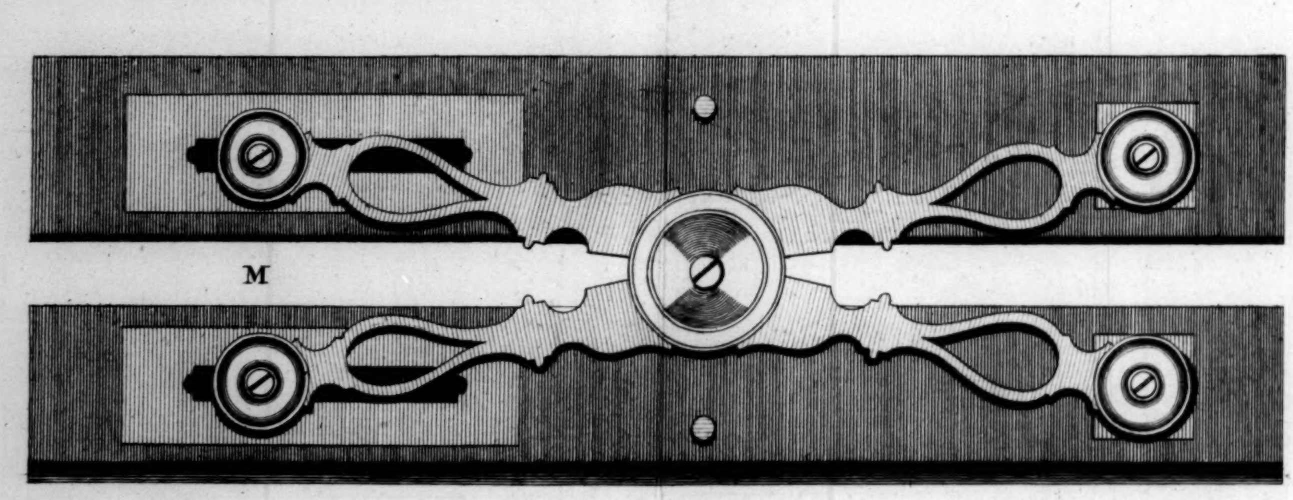

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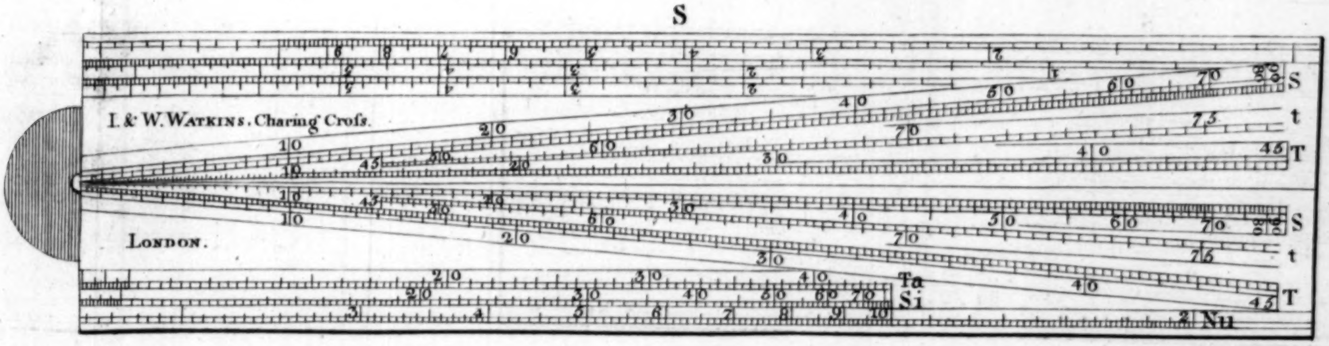
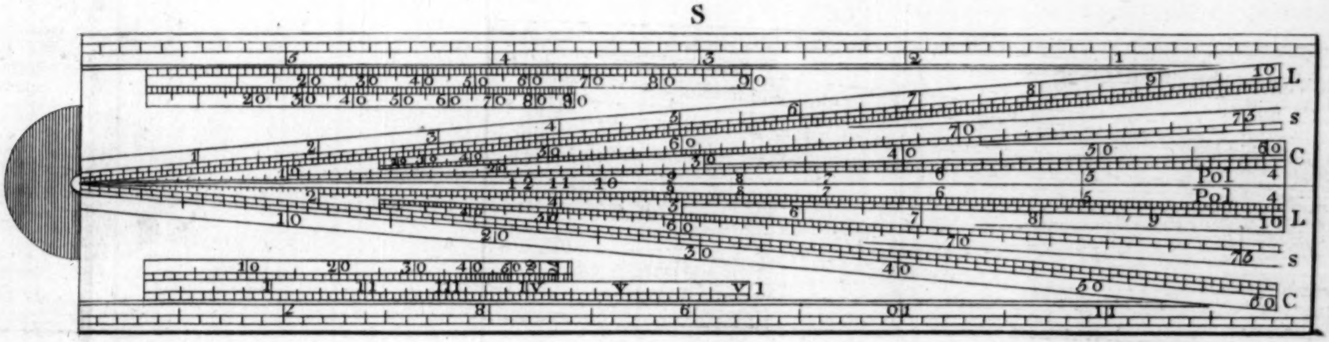
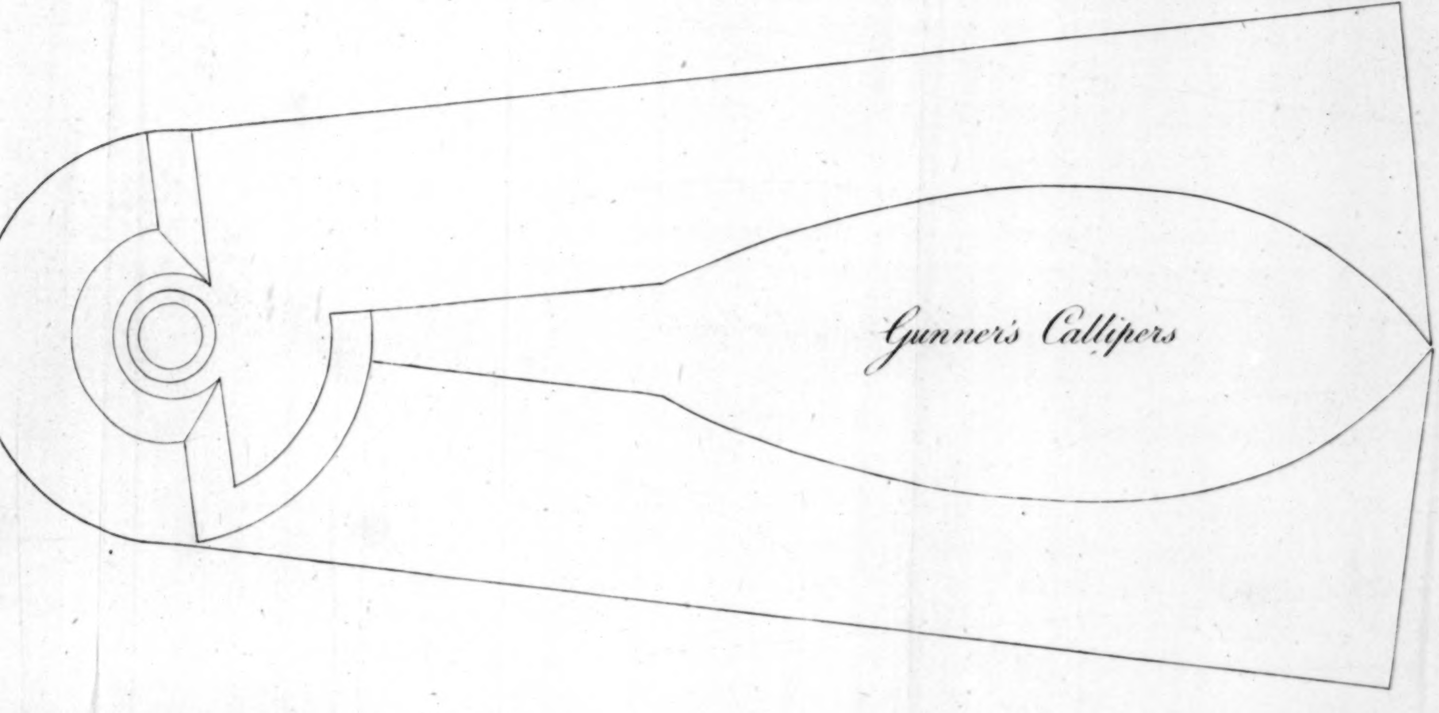
Plate II.







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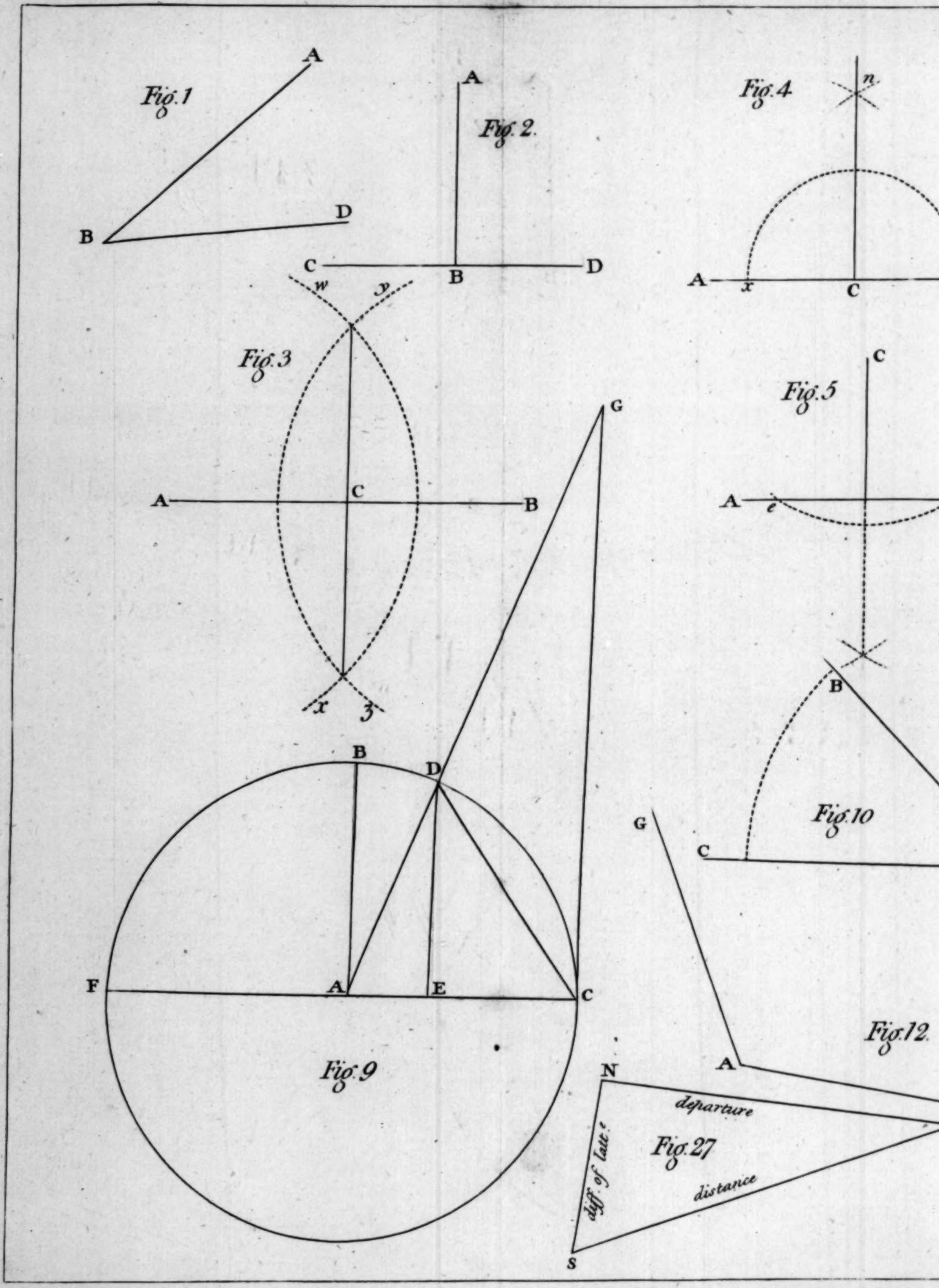

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Plate III.

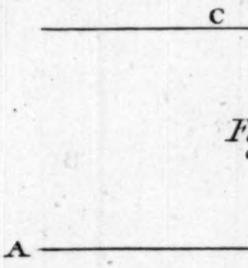
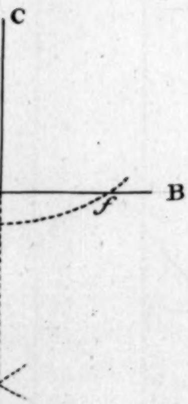


Fig. 6



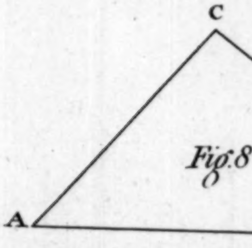
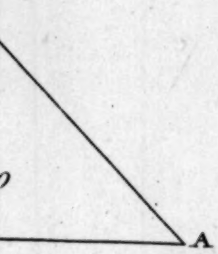


Fig. 8



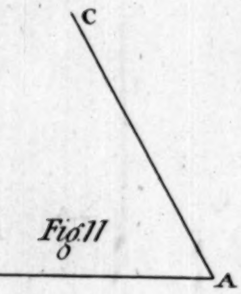


Fig. 11

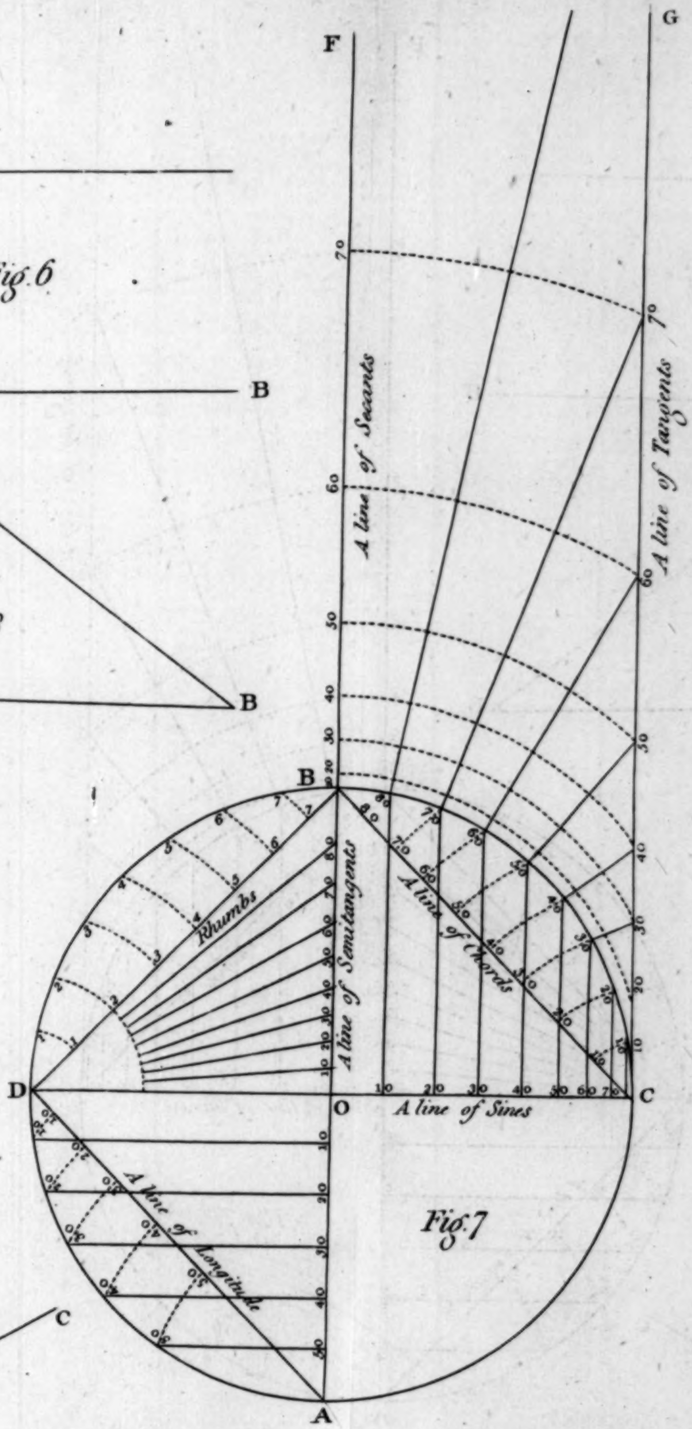
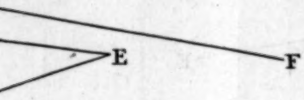


Fig. 7

Fig. 12



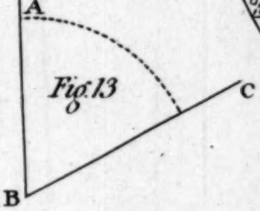


Fig. 13

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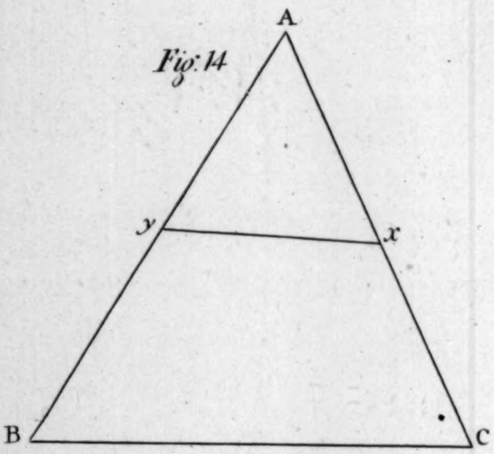
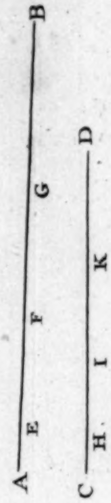
Fig. 14





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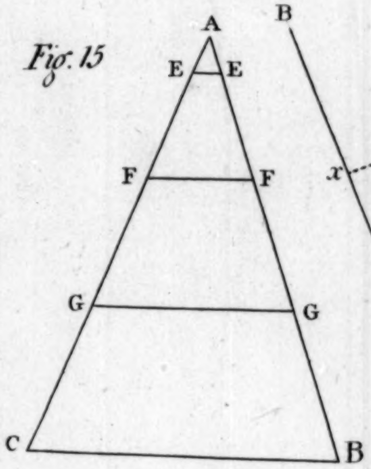


Fig. 16

Fig. 19

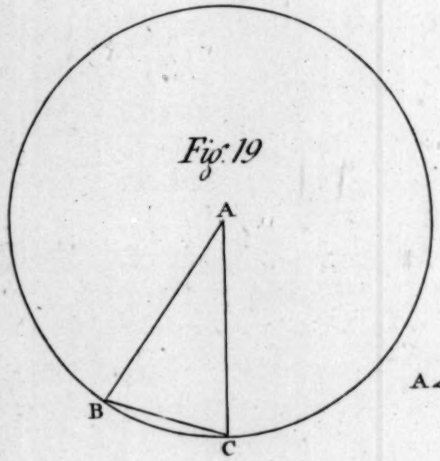


Fig. 20

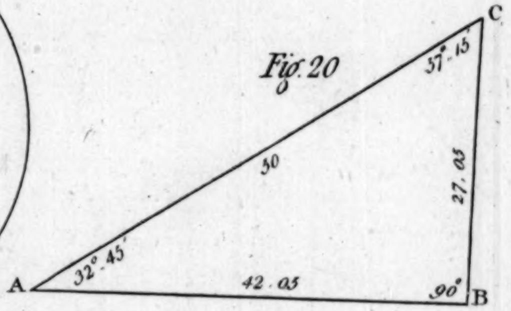
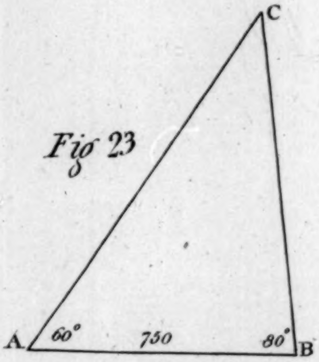


Fig 23



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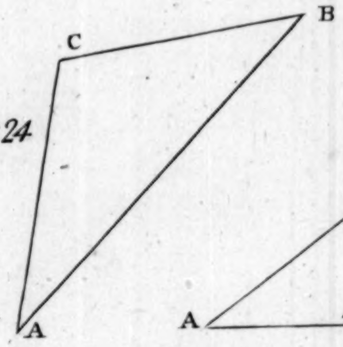

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Plate IV.

Fig. 16

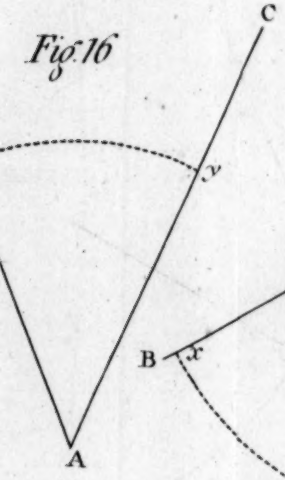


Fig. 17

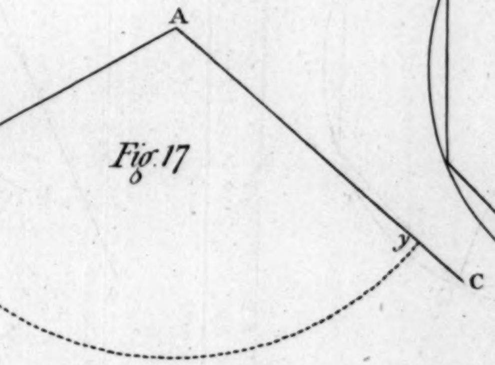
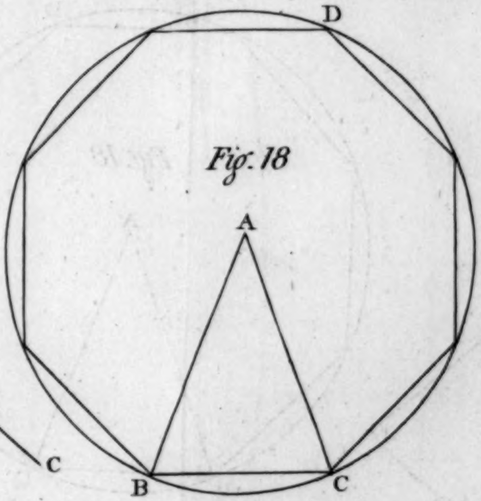


Fig. 18



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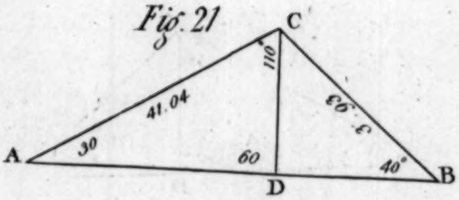

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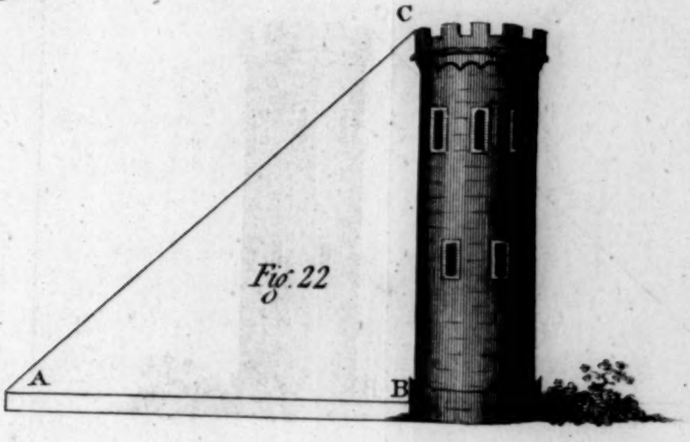
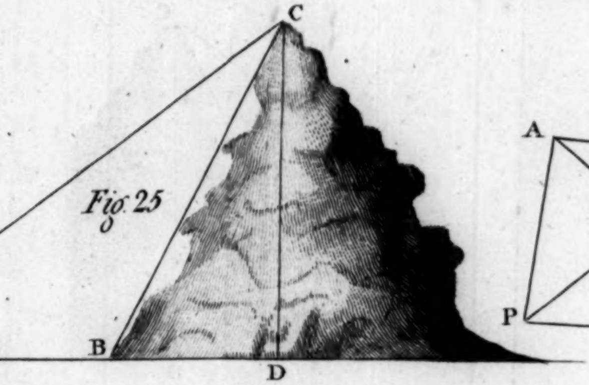


Fig. 25



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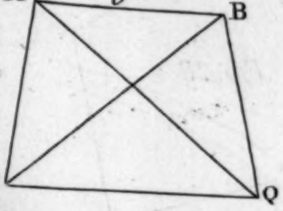

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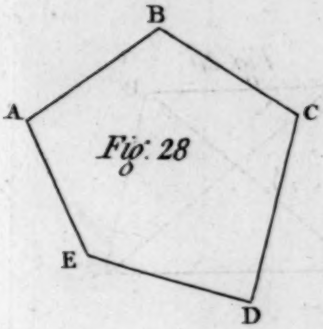

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